Albert-Ludwigs-Universität Freiburg Winter 2011/12

Particle Physics II

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Exercise sheet VIII

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In-class exercises

Exercise 45 *The Photon Mass and U(1) Local Symmetry*

Show that the kinetic term for the photon field in the Lagrange Density

$$-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

is invariant under the transformation

$$A_\mu \to A_\mu + \partial_\mu \alpha(x).$$

Show that a mass term in the Lagrange Density for the photon field:

$$\frac{1}{2}mA^\mu A_\mu$$

would not be invariant under the transformation of A_μ, thus requiring that the photon remain massless to conserve U(1) Local Symmetry.

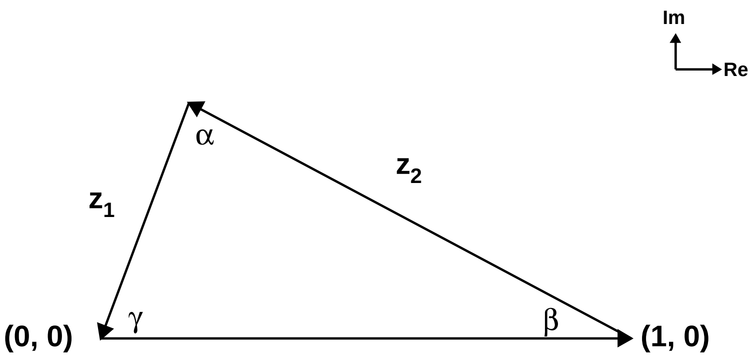
Exercise 46 *The CKM Matrix and the Unitary Triangle*

Consider the CKM matrix (which determines the relationship between the interaction eigenstates d', s', b' and the mass eigenstates d, s, b of the down-type quarks):

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}.$$

(i) Why is the following relation $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$ true?

(ii) Note that we can rewrite the relation as $1 + z_1 + z_2 = 0$ for complex numbers $z_1 = \frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$ and $z_2 = \frac{V_{td}V_{tb}^*}{V_{cd}V_{cb}^*}$, thus forming a "triangle" in the complex plane:



with angles α, β, γ defined as shown. What are the values of $\sin 2\beta$ and $\sin 2\alpha$ in terms of the complex numbers z_1 and z_2?

(iii) Express $\sin 2\beta$ and $\sin 2\alpha$ in terms of the Wolfenstein parameters A, λ, ρ, η.
(see lecture notes on the Wolfenstein parametrization of the CKM matrix)

(iv) What coordinates does the apex of the CKM unitary triangle have in the Re-Im plane, in terms of the Wolfenstein parameters?

Home exercises

Exercise 47 *Equations of Motion for the Photon Field* **6 Points**

The free Lagrangian for the photon field is given by:

$$\mathcal{L}_{\text{free}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor.

From the Euler-Lagrange equations:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu A_\nu)} \right) - \frac{\partial \mathcal{L}}{\partial A_\nu}$$

show that the equations of motion for the photon field are:

$$\partial_\mu F^{\mu\nu} = 0.$$

Show that Maxwell's equations $\nabla \cdot \vec{E} = 0$ and $-\frac{\partial \vec{E}}{\partial t} + \nabla \times \vec{B} = 0$ are reproduced when interpreting $E^i = -F^{0i}$ and $\epsilon^{ijk}B^k = -F^{ij}$.

Exercise 48 *Equations of Motion for the Scalar and Vector Fields* **5 Points**

What are the equations of motion for the scalar field Lagrangian (with self-interaction)

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4$$

and the Proca Lagrangian for a massive vector field

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}mA^\mu A_\mu?$$

What does the interaction vertex look like (Feynman diagram) for the scalar field Lagrangian?

Exercise 49 *Transformations of Vector Bosons in Yang-Mills Theory* **9 Points**

In Yang-Mills Theory, the required infinitesimal transformation rule for the three additional vector fields, to preserve SU(2) local symmetry can be given by:

$$\vec{W}_\mu \to \vec{W}_\mu - \partial_\mu \frac{\vec{\alpha}}{g} - \vec{\alpha} \times \vec{W}_\mu.$$

Note that the vector symbols do not denote a three-vector in physical space, rather denote the three vector fields $\vec{W}_\mu = W^i_\mu$ for $i = 1, 2, 3$. Note also that $\vec{\alpha} \times \vec{W}_\mu = \epsilon^{ijk}\alpha^j W^k_\mu$. (The quantity g is just a real number.)

(i) Show that the object $\partial_\mu W_\nu - \partial_\nu W_\mu$ transforms to

$$\partial_\mu W_\nu - \partial_\nu W_\mu - \vec{\alpha} \times (\partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu) + \vec{W}_\nu \times \partial_\mu \vec{\alpha} - \vec{W}_\mu \times \partial_\nu \vec{\alpha}.$$

(ii) For the object $\vec{W}_{\mu\nu} = \partial_\mu \vec{W}_\nu - \partial_\nu \vec{W}_\mu - g\vec{W}_\mu \times \vec{W}_\nu$, show that the infinitesimal transformation for $\vec{W}_{\mu\nu}$ is given by

$$\vec{W}_{\mu\nu} \to \vec{W}_{\mu\nu} - \vec{\alpha} \times \vec{W}_{\mu\nu}$$

You can make use of the identity:

$$\vec{A} \times (\vec{B} \times \vec{C}) + \vec{B} \times (\vec{C} \times \vec{A}) + \vec{C} \times (\vec{A} \times \vec{B}) = 0.$$

Note that second order terms in $\vec{\alpha}$ can be dropped.

(iii) Show therefore that kinematic term $\vec{W}_{\mu\nu}\vec{W}^{\mu\nu}$ of the Lagrange density for the three additional vector fields is invariant (under such infinitesimal transformations). Note that second order terms in $\vec{\alpha}$ can be dropped.