Particle Physics II

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Exercise sheet X

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In-class exercises

Exercise 55 *Production of Hadrons from Electron-Positron Annihilation at Low Energy*

A good experimental indication that quarks possess a colour charge comes from the measurement of the quantity

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

For $e^+e^- \to f\bar{f}$ (for any fermion f) scattering at a CM energy E_{CM} which satisfies $m_f \ll E_{\text{CM}} \ll m_Z$, one obtains the following formula for the total cross-section:

$$\sigma = \frac{\pi}{3}\left(\frac{Q_f \alpha}{E}\right)^2,$$

where Q_f is the electric charge of the fermion.

(i) Why is $R = 3\sum_i Q_i^2$, where i sums over all quark flavours below the E_{CM} threshold.

(ii) What is R for low energies (when only the three lightest quarks contribute)? What is R above the c-quark threshold and above the b-quark threshold?

Compare these results to the sheet attached.

Exercise 56 *Production of Hadrons from Electron-Positron Annihilation at the Z Pole*

Consider electron-positron annihilation at the Z pole ($E_{\text{CM}} \simeq m_Z$).
Note that quark and lepton masses are then negligible, and the photon propagator can be ignored.
In this case, the differential scattering cross-section (try it at home!) can be shown to be:

$$\frac{d\sigma}{d\Omega} = \left(\frac{g_Z^2 E}{16\pi[4E^2 - m_Z^2]}\right)^2 \left([(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2][1 + \cos^2\theta] - 8c_V^f c_A^f c_V^e c_A^e \cos\theta\right)$$

(i) Explain why at the Z pole, we have

$$R = \frac{3\sum_q [(c_V^q)^2 + (c_A^q)^2]}{[(c_V^\mu)^2 + (c_A^\mu)^2]}.$$

(ii) What is the value of R in this case? (substitute for c_V^q and c_A^q for the relevant quarks)

Exercise 57 *Theoretical Concepts and Physical Consequences*

(i) What is the difference between Abelian and non-Abelian Groups? What physical consequences are different between Abelian and non-Abelian gauge theories?

(ii) What is the reason that no colour-singlet gluon exists (the ninth gluon)? What experimental finding confirms this non-existence?

Home exercises

Exercise 58 *Hadron Collisions and Rapidity* **12 Points**

The rapidity y is a useful kinematic variable for particles produced in inelastic hadron-hadron collisions (such as $pp \to X$ at the LHC). It is usually expressed as:

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) \tag{1}$$

where $p_L = p_z$ is the longitudinal momentum along the z-axis (or scattering axis, parallel to the initial particle momentum axis) and E is the energy of the particle in the final state. Other kinematic variables for final state particles include the transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$ and the mass of the particle m.

Mathematically, the rapidity can also describe Lorentz-transformations along the scattering z-axis with $\beta = p_L/E$:

$$E' = E \cosh y - p_L \sinh y$$
$$p'_L = p_L \cosh y - E \sinh y$$

(i) Show that the relationship $\tanh y = \beta$ holds, by comparing with the standard Lorentz-transformation rules for boosts along the longitudinal z-axis.

(ii) Using the results from part (i), confirm that equation (1) holds.

(iii) How does the rapidity of a particle change under a Lorentz-boost in the z-direction (as a function of β)? Express your answer in an equation relating y' and y. How does the rapidity difference change between two particles under such a Lorentz-boost? How does the rapidity distribution of an ensemble of particles change under a Lorentz-boost?

(iv) Which scattering angle and which rapidity correspond to a longitudinal momentum $p_L = 0$? Show also that $y(-p_L) = -y(p_L)$.

(v) Show that the rapidity can also be written as

$$y = \ln \left(\frac{E + p_L}{\sqrt{p_T^2 + m^2}} \right).$$

(vi) To determine the maximum and minimum values for the rapidity y in a collision of two particles at energy $E_{CM} = \sqrt{s}$ in the centre-of-mass system, each with mass m. What is the maximum longitudinal momentum that the final state paricles can have (neglecting the mass of the particles)? Show that the maximum and minimum values of the rapidity satisfy:

$$y_{\text{max}} = -y_{\text{min}} = \frac{1}{2} \ln \frac{s}{m^2}.$$

(vii) The differential cross section for the production of hadrons for small values of p_L in the final state can be expressed as:

$$d^2\sigma = \pi F_{p_T} dp_T^2 V (dp_L/E),$$

where V is a constant, F_{p_T} varies slowly with p_T (and can be treated as a constant in integration). Find the expression for $\frac{dp_L}{dy}$. Integrate $d^2\sigma$ over p_T^2 and show that the quantity $\frac{d\sigma}{dy}$ is a constant. Draw the distribution of $\frac{d\sigma}{dy}$ as a function of y between y_{min} and y_{max}.

(viii) The average particle multiplicity in the production of hadrons $< n >$ can be obtained by integrating $\frac{d\sigma}{dy}$ over y. How does the average particle multiplicity $< n >$ depend on the centre-of-mass energy $\sqrt{s}$?

(ix) Another used quantity in hadron-collider is the pseudorapidity defined as:

$$\eta = -\ln \tan\left(\frac{\theta}{2}\right),$$

where θ is the scattering angle from the longitudinal z-axis. Show that the pseudorapidity and the rapidity are identical for massless particles.

Exercise 59 *The Weak Mixing Angle and Leptonic Asymmetry Parameters* **8 Points**

At experiments such as LEP and SLD, leptonic asymmetry parameters A_f are measured in $e^+e^- \to f\bar{f}$ processes, to obtain a determination of the weak mixing angle, more specifically $\sin^2\theta_W$. Recall that these are related using the following equations:

$$A_f = \frac{2c_V/c_A}{1+(c_V/c_A)^2}$$

$$\frac{c_V}{c_A} = 1 - 4|Q_f|\sin^2\theta_W,$$

where Q_f is the charge of the final state fermions, and c_V, c_A are the respective coefficients for the vector and axial couplings of the weak interaction.

(i) Show that the measured value of $A_\mu = 0.142 \pm 0.015$ in $e^+e^- \to \mu^+\mu^-$ events yields a value of $\sin^2\theta_W = 0.232 \pm 0.002$.

(ii) Using the value $\sin^2\theta_W = 0.232$, what can you infer about the central values of A_c and A_b, the c-quark and b-quark asymmetry factors?

(iii) What uncertainties would be required on those central values of A_c and A_b to obtain the same precision on the weak mixing angle: $\sin^2\theta_W = 0.232 \pm 0.002$?

(iv) Assume nature had instead chosen a value of $\sin^2\theta_W = 0.8$. What uncertainty would be obtained for $\sin^2\theta_W$, if one were to measure A_μ, A_c, and A_b to 0.005 uncertainty, respectively?