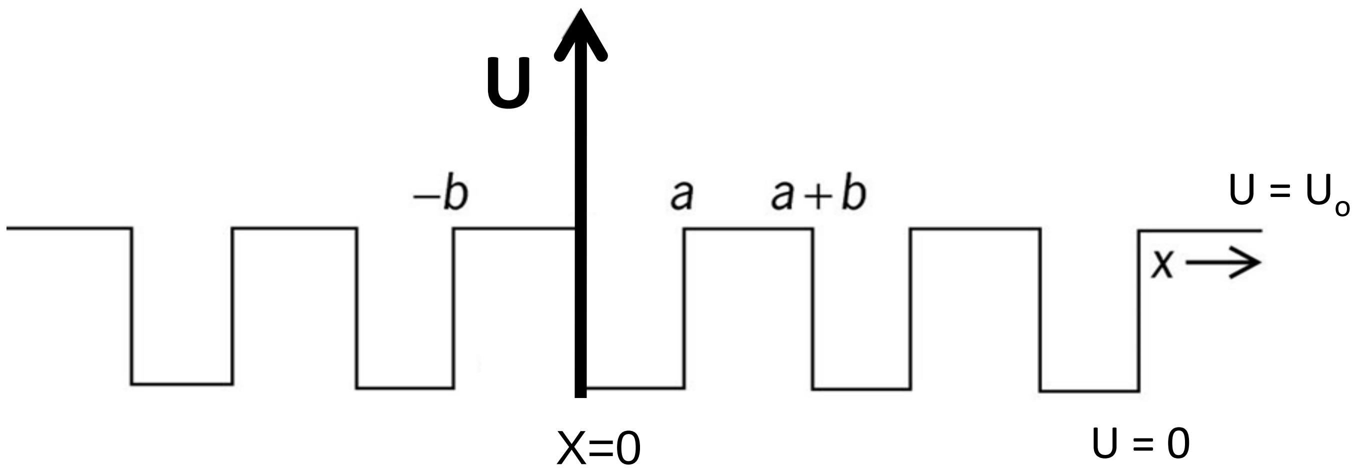
Kronig-Penney Modell



Schrödinger Gleichung

$$\hat{H}\psi(x) = E\psi(x)$$

$$(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x))\psi(x) = E\psi(x)$$

Für U(x) = 0 (0 ≤ x ≤ a) folgt:

$$-\frac{\hbar^2}{2m}\psi'' = \psi E; E = -\frac{\hbar^2 K^2}{2m}$$

$$\psi = Ae^{iKx} + Be^{-iKx} = A'\sin(Kx) + B'\cos(Kx)$$

Für U(x) = U_0:

$$-\frac{\hbar^2}{2m}\psi'' = \psi(E - U_o); E - U_o = -\frac{\hbar^2 Q^2}{2m}$$

$$\psi = Ce^{Qx} + De^{-Qx}$$

Stetigkeitsbedingung

$$\psi(x) = u_k \exp(ikx)$$

$$\psi(x+T) = u_k \exp(ikx)$$

$$\psi(a < x < a+b) = \psi(-b < x < 0)e^{ik(a+b)}$$

x = 0

$$\psi_+ = A + B$$
$$\psi_- = C + D$$
$$\Rightarrow A + B = C + D$$
$$\psi'_+ = iK[A - B]$$
$$\psi'_- = Q[-C + D]$$
$$\Rightarrow iK[A - B] = Q[-C + D]$$

x =a und x = -b

$$\psi_+ = Ae^{iKa} + Be^{-iKa}$$
$$\psi_- = Ce^{Qb} + De^{-Qb}$$
$$\Rightarrow Ae^{iKa} + Be^{-iKb} = (Ce^{Qb} + De^{-Qb})e^{ik(a+b)}$$
$$\psi'_+ = iKAe^{iKa} - iKBe^{-iKa}$$
$$\psi'_- = Q(-Ce^{-Qb} + De^{Qb})e^{ik(a+b)}$$
$$\Rightarrow ikAe^{ika} - ikBe^{-ika} = -QCe^{-Qb} + QDe^{Qb}$$

4 Gleichungen

$$A + B = C + D$$
$$iK[A - B] = Q[-C + D]$$
$$Ae^{iKa} + Be^{-iKa} = (Ce^{-Qb} + De^{Qb})e^{ik(a+b)}$$
$$ikAe^{ika} - ikBe^{-ika} = -QCe^{-Qb} + QDe^{Qb}$$

Eine Lösung wenn die Determinante verschwindet

$$\begin{pmatrix} 1 & iK & \exp(iKa) & ik\exp(ika) \\ 1 & -iK & \exp(-iKa) & -ik\exp(-ika) \\ 1 & -Q & \exp(-Qb + ik(a+b)) & -Q\exp(-Qb) \\ 1 & Q & \exp(Qb + ik(a+b)) & Q\exp(Qb) \end{pmatrix} = 0$$

$$\Longrightarrow \quad \left[(Q^2 - K^2)/2QK\right]\sinh Qb \sin Ka + \cosh Qb \cos Ka = \cos k(a+b)$$