Particle Physics II

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Exercise sheet V

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In-class exercises

Exercise 31 *Vertex Corrections and the Magnetic Moment*

The interaction of an electron with an electromagnetic field A_μ has a vertex correction due to next-to-leading order Feynman diagrams. In particular, for small momentum transfer q^2, this is given by:

$$e\bar{u}_f\left\{\gamma_\mu\left[1+\frac{\alpha}{3\pi}\frac{q^2}{m_e^2}\left(\ln\frac{m_e}{m_\gamma}-\frac{3}{8}\right)\right]-\frac{\alpha}{2\pi}\frac{1}{2m_e}i\sigma_{\mu\nu}q^\nu\right\}u_i,$$

where m_e is the mass of the electron and m_γ is the mass of the virtual photon.

Using the Gordon identity

$$e\bar{u}_f\gamma^\mu u_i=\frac{e}{2m_e}\bar{u}_f\left[(p_f^\mu+p_i^\mu)+i\sigma^{\mu\nu}(p_{\nu,f}-p_{\nu,i})\right]u_i,$$

and equate the term proportional to $i\sigma_{\mu\nu}q^\nu$ to the magnetic moment $\vec{\mu}$ of the electron. Show then that the vertex correction yields the following relation for the gyromagnetic ratio of the electron:

$$\frac{g-2}{2}=\frac{\alpha}{2\pi}.$$

Home exercises

Exercise 32 *Running Coupling Constants in QED* **6 Points**

The dependency on the electromagnetic coupling constant α_{EM} on the squared momentum transfer q^2 is given by:

$$\alpha_{\text{EM}}(q^2) = \frac{\alpha_{\text{EM}}(\mu^2)}{1 - \Pi_{\text{EM}}(q^2, \mu^2)},$$

where μ^2 is a scalar parameter, and $\Pi_{EM}(q^2, \mu^2 = 0)$ is given by:

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \sum_{2m_f < |q|} N_c Q_f^2 \frac{\alpha}{3\pi}(\ln \frac{q^2}{m_f^2} - \frac{5}{3}).$$

Here, N_c stands for the number of different colours for the different fermion species ($N_c = 1$ for fermions while $N_c = 3$ for quarks). The index f runs over the different fermions with charge Q_f, and the summation occus for those fermions species with masses that satisfy the condition $2m_f < |q|$ (for which pair creation is possible at the given momentum transfer).

In addition, α is the electromagnetic coupling constant in the low energy limit:

$$\alpha = \alpha_{\text{EM}}(q^2 = 0, \mu) \simeq \frac{1}{137}.$$

(i) For momentum transfers that satisfy $2m_b < |q| < 2m_t$, show that

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \frac{\alpha}{3\pi}(3 + R) \ln \frac{q^2}{m_0^2},$$

where $R = N_c \sum_{f=1}^{5} Q_f^2$ and $m_0 = 0.30$ GeV is the effective mean of all fermion masses in question.

(ii) Compare the value of α_{EM} for momentum transfer $q^2 = M_Z^2$ to the low energy limit α.

(iii) What value does R take for momentum transfers that allow top quark pair production?

(iv) At what momentum transfer does α_{EM} diverge?
(Note for higher momentum transfers, $m_0 = 0.94$ GeV.)

Exercise 33 *Running Coupling Constants in QED II* **5 Points**

The OPAL Experiment at the Large Electron-Positron Collider measured the dependency of the coupling constant α_{EM} on the momentum transfer in Bhabha Scattering ($e^+e^- \to e^+e^-$) for very small scattering angles (scattering in the forward direction). The following momentum transfer ranges were investigated:

$$1.81\text{GeV}^2 \leq -t \leq 6.07\text{GeV}^2.$$

Recall that the unpolarized squared scattering amplitude for this process is given by:

$$|\mathcal{M}|^2 = 2e^4 \Big(\frac{u^2 + t^2}{s^2} + \frac{2u^2}{st} + \frac{u^2 + s^2}{t^2} \Big).$$

(i) Show that the t-channel contribution dominates the scattering process for small angle scattering.

(ii) Calculate the effective change of the electromagnetic coupling constant that was measured in the momentum transfer ranges observed:

$$\Delta\Pi_{\text{EM}} = \Pi_{\text{EM}}(t_{max}) - \Pi_{\text{EM}}(t_{min}).$$

Recall the relation from the previous problem:

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \frac{\alpha}{3\pi}(3 + R) \ln \frac{q^2}{m_0^2}.$$

Exercise 34 *Two Body Decay Kinematics* **3 Points**

Consider a two body decay: $A \rightarrow BC$, in the rest frame of particle A.

Show that the following equations hold for the energy of decay particles B and C.

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} \qquad E_C = \frac{m_A^2 - m_B^2 + m_C^2}{2m_A}.$$

Exercise 35 *Tritium Decay and Neutrino Masses* **7 Points**

The Karlsruhe Tritium Neutrino Experiment (KATRIN) investigates weak decays of Tritium

$$^3\mathrm{H} \rightarrow ^3\mathrm{He}^+ + e^-\bar{\nu}_e$$

in order to determine the mass of electron anti-neutrino $\bar{\nu}_e$.

(i) For a three body decay $A \rightarrow B + C + D$, what is the energy E_B of particle B as a function of the mass m_A of particle A and the invariant mass M_{CD} of the combined system of particles C and D?

(ii) In what scenario is the energy of particle B at its maximum, and what scenario for its minimum? What are the corresponding values for E_B^{max} and E_B^{min}?

(iii) Given $m_e = 0.510999$ MeV, $m_{^3\mathrm{H}} = 2809$ MeV, and $\Delta(m_{^3\mathrm{H}}, m_{^3\mathrm{He}}) = 0.5296$ MeV, what is the maximum energy of the electron assuming the neutrinos are massless? What is the maximum kinetic energy of the electron?

(iv) For a given measured maximum electron energy m_e^{max}, how large is the mass of the emitted anti-neutrinos?