Particle Physics II

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Exercise sheet IX

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In-class exercises

Exercise 50 *Measurement of the forward-background asymmetry at PETRA*

The Positron-Electron Tandem Ring Accelerator PETRA was a particle accelerator at DESY in Hamburg, studying electron-positron collisons from 1978-1986 at center-of-mass energies of up to 46 GeV. It allowed the measurement of the forward-background asymmetry A_{FB} in $e^+e^- \to \mu\mu$ events.

From the lectures, the total cross section for the process $e^+e^- \to f\bar{f}$ is known as

$$\sigma_{\rm tot}(s) = \frac{N_C \pi}{s}\frac{4}{3}\left[\alpha^2 Q_f^2 - 8\alpha Q_f \Re(\chi) c_{v_e} c_{v_f} + 16\,|\chi|^2 \left(c_{v_e}^2 + c_{a_e}^2\right)\left(c_{v_f}^2 + c_{a_f}^2\right)\right]$$

The following relations apply:

$$\begin{aligned} c_{v_i} &= I_{3,i} - 2Q_i \sin^2\theta_w, \\ c_{a_i} &= I_{3,i} \\ \chi(s) &= \frac{G_F m_Z^2}{8\pi\sqrt{2}} \frac{s}{s - m_Z^2 + i\frac{s\Gamma_Z}{m_Z}}. \end{aligned}$$

Also known is the general expression for A_{FB} for these type of events as

$$A_{\rm FB} = \frac{\sigma_{\rm F} - \sigma_{\rm B}}{\sigma_{\rm F} + \sigma_{\rm B}}.$$

with

$$\sigma_{\rm F} - \sigma_{\rm B} = \frac{N_C \pi}{2s}\left[-8\alpha Q_f \Re(\chi)\cdot 2\cdot C_{a_e} C_{a_f} + 16\,|\chi|^2 \cdot 8 C_{v_e} C_{v_f} C_{a_e} C_{a_f}\right].$$

Show that the forward-background symmetry for the case $f = \mu$ is approximately

$$A_{\rm FB} = -\frac{3G_F}{\sqrt{2}e^2} C_A^2 s.$$

and interprete the result (Fig. 1).

Hint: Do the following step-by-step (using $\sqrt{s} \ll M_Z$):

(i) Simplify $\mathrm{Re}(\chi)$ for $\sqrt{s} \ll M_Z$.

(ii) Show that for $\sigma_{\rm F} - \sigma_{\rm B}$, the $\gamma^* - Z$ interference term dominates and the rest can be neglected.

(iii) Show that the γ^* term dominates for the total cross section.

(iv) Combine the results of these steps to express A_{FB}.

(v) Fill in Q_F of the muon, α, and use $C_{Ae} = C_{A\mu}$.

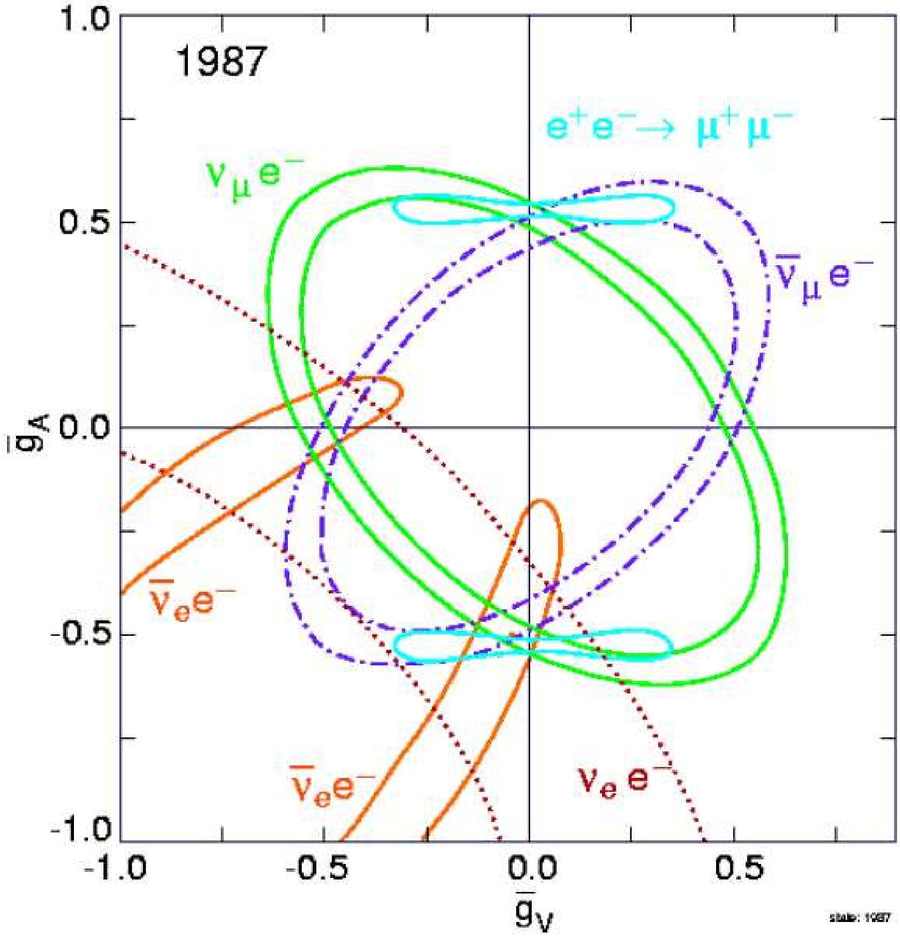

Figure 1: Different measurements of the couplings of leptons to Z bosons, $g_V = C_V$ and $g_A = C_A$.

Home exercises

Exercise 51 *Total cross section $e^+e^- \to f\bar{f}$* **6 Punkte**

Using the equation from the in-class exercises, and the values of the Z mass, the total width of the Z boson, Γ_Z, the Fermi coupling constant G_F and the weak mixing angle θ_w, determine numerically the total cross section for $e^+e^- \to q\bar{q}$ processes ($q = u, d, s, c, b, t$) as well as the fraction $\frac{\sigma_i}{\sigma_{\text{tot}}}$ of the contributions to σ_{tot}: via γ^* exchange, Z-γ^* interference, and Z exchange. Do this for the following center-of-mass energies:

$$\sqrt{s} = 10\,\text{GeV},\; 30\,\text{GeV},\; 60\,\text{GeV},\; m_Z \text{ und } 200\,\text{GeV}.$$

$$\begin{aligned} m_Z &= (91.1875 \pm \text{ GeV} \\ \Gamma_Z &= 2.4952\,\text{GeV} \\ G_F &= 1.166 \times 10^{-5}\,\text{GeV}^{-2} \\ \sin^2\theta_w &= 0.234 \end{aligned}$$

Exercise 52 *Total Z boson width* **6 Punkte**

The partial Z boson width for each of the neutrino pairs is given by

$$\Gamma(Z \to \nu\nu) = \frac{G_F M_Z^3}{12\pi\sqrt{2}}$$

and measured as $\Gamma(Z \to \nu\nu) = 167$ MeV. For $Z \to ee$, the contributions are given by

$$\Gamma(Z \to e_R^+ e_L^-) = (2C_L)^2 \Gamma(Z \to \nu\nu)$$

and a similar contribution for a pair with a right-handed e^- (remember that left-handed antiparticles normally behave like right-handed particles). This is also approximately valid for other lepton flavor pairs.

The contribution from allowed decays to a quark pair with any given quantum numbers is the same as for leptons.

Which electron helicity states are allowed (i.e., unsuppressed) for $Z \to ee$ decays and why?
What is the total Z boson width? Compare to the literature value $\Gamma_Z = 2.4952$.

Exercise 53 *Total W boson width* **4 Punkte**

The partial W boson widths for decays to electron+neutrino is given by

$$\Gamma(W \to e\bar{\nu}_e) = \frac{G_F M_W^3}{6\pi\sqrt{2}}$$

The same relation approximately also holds for other lepton flavours; and similarly for any quark pair with given quantum numbers,

$$\Gamma(W \to qq') = \frac{G_F M_W^3}{6\pi\sqrt{2}} |V_{qq'}|^2,$$

$\Gamma(W \to \ell\nu)$ has been measured to be 227 MeV. What is the value of the total W boson width, and what is the corresponding branching ratio $\text{BR}(W \to \ell\nu)$? You can approximate V as diagonal.

Exercise 54 *Number of neutrino flavors* **4 Punkte**

(i) The total decay width of the Z boson is given by:

$$\Gamma_Z = \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\text{had}} + \Gamma_{\text{inv}}.$$

Here, Γ_l with $l = e, \mu, \tau$ are the partial widths of decays to lepton pairs $l\bar{l}$, Γ_{had} the partial width to decays to quark pairs, and Γ_{inv} to invisible particles (like neutrinos).
Show that the ratio of invisible and leptonic decay width is given by

$$R_{\text{inv}} = \frac{\Gamma_{\text{inv}}}{\Gamma_l} = \left[\frac{12\pi R_l}{\sigma^{\text{had}}_{\text{peak}} M_Z^2}\right]^{\frac{1}{2}} - R_l - 3.$$

Here, $R_l = \frac{\Gamma_{\text{had}}}{\Gamma_l}$ and $\sigma^{\text{had}}_{\text{peak}}$ is the cross section for $Z \to$ hadrons for $\sqrt{s} = m_Z$.
You can assume lepton universality; the peak cross section is given by $\sigma^{\text{had}}_{\text{peak}} = \frac{12\pi\Gamma_e\Gamma_{\text{had}}}{M_Z^2\Gamma_Z^2}$.

(ii) The theoretical prediction for the Standard modell is

$$R_{\text{inv}} = N_\nu \left(\frac{\Gamma_\nu}{\Gamma_l}\right)_{\text{SM}}.$$

with

$$\left(\frac{\Gamma_{\nu_l}}{\Gamma_l}\right)_{\text{SM}} = 1.99125.$$

Show that this implies that the Standard model contains $N_\nu = 3$ light neutrino flavors and calculate the uncertainty on this number (using the uncertainties on the LEP measurements given below). The equations above imply that for larger $\sigma^{\text{had}}_{\text{peak}}$, the predicted number of light neutrino flavors becomes smaller (see Fig. 2).

$$m_Z = (91.1875 \pm 0.0021)\,\text{GeV}$$
$$R_l = 20.767 \pm 0.025$$
$$\sigma^{\text{had}}_{\text{peak}} = (41.540 \pm 0.037)\,\text{nb}$$

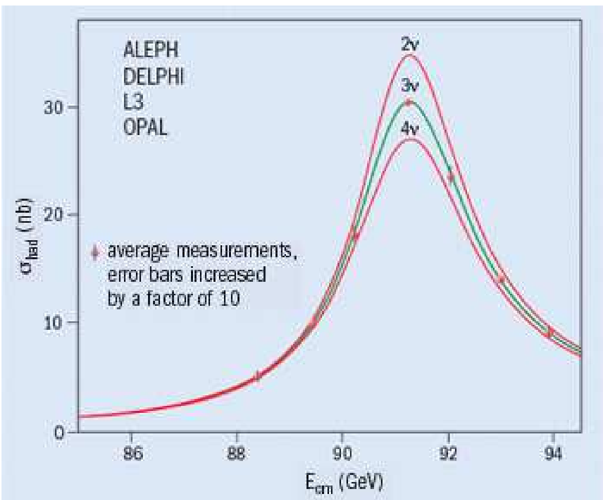

Figure 2: σ(had) as a function of the center-of-mass energy for different numbers of light neutrino flavors.