Particle Physics II

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Exercise sheet XI

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In-class exercises

Exercise 60 *Deep inelastic scattering in the quark-parton-model*

The results of deep inelastic ep scattering can be interpreted as elastic scatter processes of the electron with the partons contained in the proton. The cross section of elastic scattering of an electron off a point-like particle at rest (charge q, spin $\frac{1}{2}$) is given by:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 q^2}{4E^2 \sin^4 \frac{\theta}{2}} \frac{E'}{E} \left(\cos^2 \frac{\theta}{2} + \frac{Q^2}{2M^2} \sin^2 \frac{\theta}{2} \right). \quad (1)$$

Here, E and E' are the energies of the incoming and outgoing electron, respectively; θ the scattering angle and $Q^2 = -t$ the virtuality (momentum transfer to the proton).

Explain the meaning of the different terms in equation (1). How does the cross section change for the case of spin-less target particles? How for very heavy target particles?

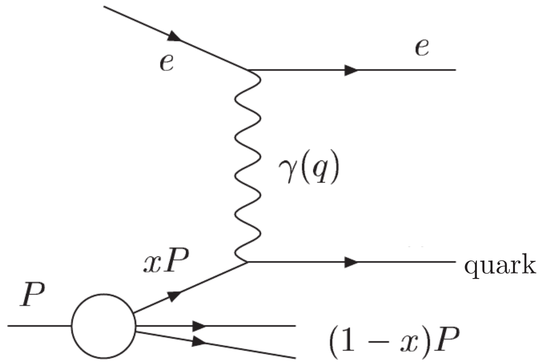
Exercise 61 *Quark-(anti)-quark interaction*

Consider the interaction of a quark with an (anti-)quark. Compute the color factor for the case of a quark and an anti-quark in the following color octet state: $B\bar{G}$.

Home exercises

Exercise 62 *Kinematics of deep inelastic scattering (DIS)* **6 points**

Deep inelastic ep scattering can be interpreted as elastic scattering of an electron off a proton.



The parton carries a fraction x of the proton momentum P. The momenta of incoming and outgoing electrons are k und k', and the momentum transfer is $q = k - k'$.

(i) Show that x is given by

$$x_{\text{BJORKEN}} = \frac{-q^2}{2P \cdot q}$$

if the transverse momentum of the parton, the parton mass, the electron mass, and the proton mass are negligible.

(ii) Show that the LORENTZ invariant $\nu = \frac{P \cdot q}{M}$ equals the energy transfer $\tilde{\nu} = E - E'$ of the electron in the rest frame of the proton.

(iii) Figure 1 shows a typical DIS event, recorded with the ZEUS detector at DESY. The positron enters from the left with an energy of 27.5 GeV, the proton from the right with an energy of 820 GeV. The polar angle at ZEUS is measured from the direction of the proton beam. In the recorded event, the electron is scattered to $\theta_e = 39.3\,^\circ$ and deposits $E'_e = 166\,\text{GeV}$ in the electromagnetic calorimeter.

The following LORENTZ invariants can be used for the kinematic description of the events:

$$x = \frac{-q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot k} \qquad s = (k+P)^2 \qquad Q^2 = -q^2$$

Derive a relation between Q^2, x, y and s. s is a constant for the given process, so two degrees of freedom remain. Neglect all particle masses.

Compute x and Q^2 for this event.

Hint: Compute x from y and Q^2.

Exercise 63 *Deep inelastic scattering in the quark-parton-model* **6 points**

(i) The elastic scattering of electrons off protons is described by the ROSENBLUTH equation:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16M^2E^2\sin^4\frac{\theta}{2}}\frac{E'}{E}\left(K_2(Q^2)\cos^2\frac{\theta}{2} + 2K_1(Q^2)\sin^2\frac{\theta}{2}\right). \tag{2}$$

The quantities $K_1(Q^2)$ and $K_2(Q^2)$ are the form factors known from the lectures. They depend on the charge distribution and the magnetic dipole moment via FOURIER transformation. Compare equations (1) and (2) to get the form factors of point-like particles.

(ii) Inelastic scatter processes of electrons off protons are described by the differential cross section

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2\sin^4\frac{\theta}{2}}\left(W_2(Q^2, x)\cos^2\frac{\theta}{2} + 2W_1(Q^2, x)\sin^2\frac{\theta}{2}\right). \tag{3}$$

The structure functions $W_1(Q^2, x)$ and $W_2(Q^2, x)$ depend on Q^2 and the BJORKEN variable x. Typically, experiments measure E' and θ. For the theoretical descriptions, the LORENTZ invariants

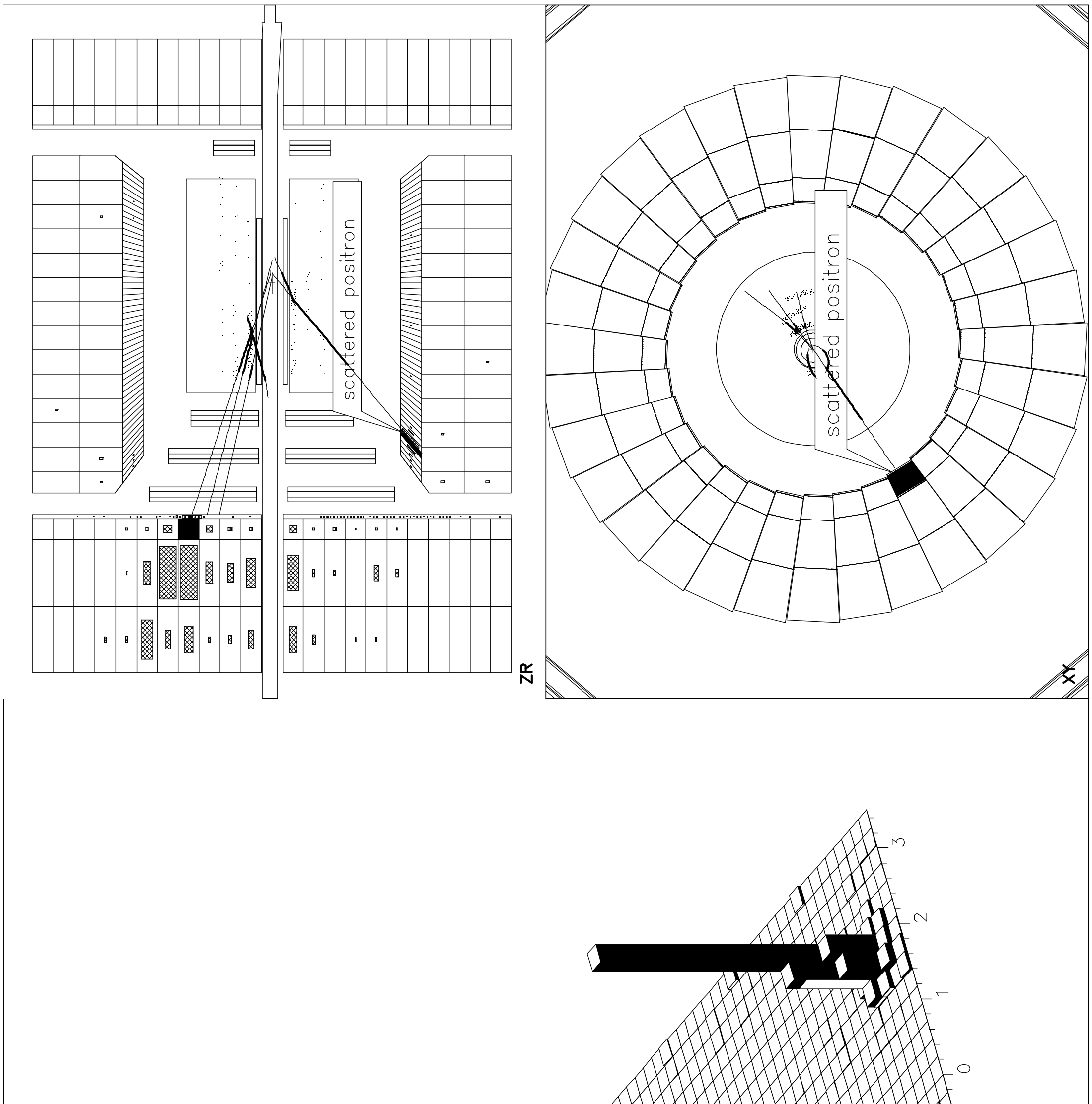


Figure 1: A DIS event recorded by ZEUS, in the r-z projection (upper left) and r-ϕ projection (upper right) as well as in the η-ϕ plane. The polar angle at ZEUS is measured with respect to the direction of the proton beam (coming from above in the upper left plot) and defines in good approximation the LORENTZ-invariant pseudorapidity $\eta = -\ln\tan\left(\frac{\theta}{2}\right)$.

Q^2 and x (or ν) are preferred. Assume the proton consists of Quarks i with masses m_i, where the quarks carry the fraction z_i of the total momentum p of the proton.
Derive an expression for the structure function $W^i_{1,2}$.
The ansatz

$$W_{1,2}(Q^2,\, x) = \frac{K_{1,2}(Q^2,\, x)}{2MQ^2}\delta(x-1) \tag{4}$$

for the structure function $W_{1,2}$ and equation (3) lead to the ROSENBLUTH equation — $x = 1$ thus corresponds to elastic scattering. Which relations exist between the m_i and the total mass M, and between x_i and x?

(iii) The probability distributions of the momentum fractions z_i are typically named $f_i(z_i)$. Derive an expression for W_1 and W_2 by integrating over z_i and summing over all quarks of the proton.

(iv) Experiments show that the dimensionless structure functions

$$F_1(x) = MW_1(Q^2,\, x) \qquad \text{und} \qquad F_2(x) = \nu W_2(Q^2,\, x) \tag{5}$$

only depend on the BJORKEN variable x (at leading order). This phenomenon is known as BJORKEN scaling. How can it be explained in the context of the quark-parton-model? Which relation between F_2 und F_1 do you expect, considering that all quarks carry spin $\frac{1}{2}$? This is known as the CALLAN-GROSS relation. How does the relation change for spin-less partons in the proton? Experiments deliver overwhelming evidence for the validity of $\frac{xF_1}{F_2} = \frac{1}{2}$.

Exercise 64 *Quark-(anti)-quark interaction* **3 points**

Consider the interaction of a quark with an (anti-)quark. Compute the color factor for the case of a quark and an anti-quark in the following color octet state: $R\bar{B}$.

Exercise 65 *Determination of α_s in τ lepton decays* **5 points**

During the lectures it was shown that the ratio R_τ of the hadronic and leptonic decay width of τ leptons provides evidence for the existence of color charge.

(i) Which naive value for $R_\tau = \frac{\mathrm{BR}(\tau\to\mathrm{hadrons})}{\mathrm{BR}(\tau\to e\,\nu_e\,\nu_\tau)}$ do you get at tree level if CABBIBO-mixing is neglected in the quark sector, and mass differences of final state particles are neglected?

(ii) Experimentally the value for R_τ is 20 % larger than the naive prediction from (i). Explain the difference qualitatively with higher-order corrections.

(iii) The measured discrepancy from the prediction at tree level can be used to determine the strong coupling constant $\alpha_s(M_\tau^2)$. Show that the R_τ can be calculated from the branching ratio $\mathrm{BR}(\tau \to e\,\nu_e\,\nu_\tau)$. Use $\mathrm{BR}(\tau \to \mu\,\nu_\mu\,\nu_\tau) = 0.97\,\mathrm{BR}(\tau \to e\,\nu_e\,\nu\,_\tau)$. The factor 0.97 takes the mass difference between electron and muon into account.

(iv) Considering higher-order correction, R_τ is given as

$$R_\tau = 3\cdot(1+\delta_{\text{ew. }1})(1+\delta_{\text{ew. }2}+\delta^{\text{pert.}}_{\text{QCD}}+\delta^{\text{non-pert.}}_{\text{QCD}}).$$

Here, electro-weak corrections take on the values $\delta_{\text{ew. }1} = 0.0194$ and $\delta_{\text{ew. }2} = 0.0010$, and the non-perturbative correction is $\delta^{\text{non-pert.}}_{\text{QCD}} = -0,007 \pm 0,004$. The perturbative QCD corrections up to three loops are given by:

$$\delta^{\text{pert.}}_{\text{QCD}} = \frac{\alpha_s}{\pi} + 5,2\left(\frac{\alpha_s}{\pi}\right)^2 + 26,4\left(\frac{\alpha_s}{\pi}\right)^3.$$

The branching ratio $\mathrm{BR}(\tau \to e\,\nu_e\,\nu_\tau)$ has been experimentally determined to be $(17,84 \pm 0,06)\,\%$. Use this information to calculate α_s.