Albert-Ludwigs-Universität Freiburg Winter 2014/15

Particle Physics II

Markus Schumacher, Anna Kopp, Stan Lai

Problem Set I

28 October 2014

Please use $c = 3.00 \times 10^8\,\mathrm{m/s}$ and $\hbar c = 1.97 \times 10^{-7}\,\mathrm{eV} \cdot \mathrm{m}$ for all numerical computations.

In-class exercises

Exercise 1 *Scattering Kinematics*

(a) Consider a scattering process of two particles with centre-of-mass energy $E_{CM} = \sqrt{s}$. Show that the incident momentum $\vec{p}_i^*$ of any of the two particles in the centre-of-mass frame satisfies

$$|\vec{p}_i^*|^2 = \frac{1}{4s}\left[s - (m_1 + m_2)^2\right]\left[s - (m_1 - m_2)^2\right],$$

for particle masses m_1 and m_2. Start by writing $\sqrt{s} - E_1 = E_2$, and square both sides, and then substitute for the energy terms, using the relativistic relation $E^2 = |\vec{p}|^2 + m^2$.

(b) How can you tell that this expression of $|\vec{p}_i^*|$ is Lorentz invariant?

(c) In some cases you can neglect the masses of a particle or particles, if they are much smaller than the collision energy. In the case of electron-proton collisions, one often neglects the mass of the electron. In this particular case, show that the expression for $\vec{p}_i^*$ simplifies to:

$$|\vec{p}_i^*|^2 = \frac{E_e^2 m_p^2}{s}$$

whereby s is the Mandelstam variable which can be written $s = (p_1 + p_2)^2$ for incident particles 1 and 2, and E_e is the energy of the electron in the proton rest frame.

Exercise 2 *Mandelstam Variables*

Consider the $2 \to 2$ scattering process $1 + 2 \to 3 + 4$.

(a) For the case of identical mass ($m_1 = m_2 = m_3 = m_4 \equiv m$) in the center-of-mass system, show that

$$s = 4\left(|\vec{p}|^2 + m^2\right), \qquad t = -2|\vec{p}|^2\left(1 - \cos\theta\right), \qquad u = -2|\vec{p}|^2\left(1 + \cos\theta\right),$$

where $|\vec{p}| = |\vec{p}_i| = |\vec{p}_f|$ is the modulus of the 3-momentum of the incoming and outgoing particles, and $\theta = \angle\left(\vec{p}_1, \vec{p}_3\right)$ is the scattering angle.

(b) For which scattering angle do t and u reach their minima / maxima?

Exercise 3 *Fun with Natural Units*

(a) Determine the value of the gravitational constant $G_N \approx 6.7 \times 10^{-11}\,\frac{\mathrm{m}^3}{\mathrm{kg\,s}^2}$ in natural units of particle physics. (in terms of eV)

(b) Express the Planck mass $M_P = \sqrt{1/G_N}$ in natural units of particle physics.

(c) In SI units, the electron mass is given by 9.11×10^{-31} kg. Express this in units of MeV.

Homework

Exercise 4 *More Fun with Natural Units* **3 Points**

(a) In 2012, a Higgs boson was found at the LHC experiments with a mass of around 125 GeV. Express this mass in SI units.

(b) The muon lifetime is measured quite precisely and is $\tau_\mu = 2.2\ \mu$s. Express this in terms of units in eV.

Exercise 5 *Electron-Proton Elastic Scattering* **8 Points**

Consider elastic electron-proton scattering, where the incoming electron (1) has incident energy E_1, and the proton (2) is initially at rest with $\vec{p}_2 = 0$. Label the outgoing electron and proton with (3) and (4), respectively, with the angle between the momentum of the outgoing electron and the original incoming momentum denoted by θ. Assume for this problem that the electron mass is negligible ($m_e \simeq 0$).

(a) Draw a sketch of the scattering process in the laboratory frame.

(b) Write down the 4-vectors of the incoming and outgoing particles in terms of energies, masses, and the angle θ.

(c) Express the quantity E_3 in terms of E_1, m_p, and θ, where m_p is the proton mass.

(d) What is the expression of the differential cross-section $\frac{d\sigma}{d\Omega}$ for this process? Use the fact that

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{dt}\left|\frac{dt}{d\Omega}\right|$$

for the Mandelstam variable $t = (p_1 - p_3)^2 = (p_2 - p_4)^2$, and the Lorentz invariant form of the scattering cross-section for $2 \to 2$ processes (with distinct particles)

$$\frac{d\sigma}{dt} = \frac{|\mathcal{M}_{fi}|^2}{64\pi s |\vec{p}_i^{\,*}|^2},$$

whereby $\vec{p}_i^{\,*}$ is the momentum of the incident electron (see Problem 1) in the centre-of-mass frame and s is the Mandelstam variable equivalent to the square of the centre-of-mass energy.

Exercise 6 *Lorentz Invariant Flux Factor* **4 Points**

From the lectures, the cross-section for the scattering process $A + B \to 1 + 2$ can be expressed as

$$\sigma = \frac{1}{(2\pi)^2 F}\int |\mathcal{M}_{fi}|^2 \delta(E_A + E_B - E_1 - E_2)\delta^3(\vec{p}_A + \vec{p}_B - \vec{p}_1 - \vec{p}_2)\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{d^3\vec{p}_1}{(2\pi)^3 2E_1},$$

where the momenta of the incoming particles A and B are anti-parallel with respect to each other (i.e. $\vec{v}_A \cdot \vec{v}_B = -|\vec{v}_A||\vec{v}_B|$). Here, the Lorentz invariant flux factor is given by $F = 4E_A E_B |\vec{v}_A - \vec{v}_B|$ for incoming particle velocities $\vec{v}_A$ and $\vec{v}_B$. Show that the flux factor F is indeed Lorentz invariant.

Hint: Show that F can be written as $4\sqrt{(p_A \cdot p_B)^2 - m_A^2 m_B^2}$ for 4-vectors p_A and p_B.

Exercise 7 *Relativistic kinematics for the two-body decay* **5 Points**

Consider the decay $A \to BC$ in the rest frame of particle A.

(a) Show that the following equation describes the energies of the outgoing particles as a function of the masses involved:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_A} \qquad E_C = \frac{m_A^2 - m_B^2 + m_C^2}{2m_A}$$

(b) Using this result, show that the momentum of the outgoing particles $|\vec{p}_f| = |\vec{p}_B| = |\vec{p}_C|$ is given by:

$$|\vec{p}_f| = \frac{1}{2m_A}\sqrt{m_A^4 + m_B^4 + m_C^4 - 2m_A^2 m_B^2 - 2m_A^2 m_C^2 - 2m_B^2 m_C^2} \quad .$$

(c) What is the energy of a μ^- produced in the decay $\pi^- \to \mu^- \bar{\nu}$ in the pion rest frame? You can use $m_\pi = 140$ MeV, $m_\mu = 106$ MeV, and $m_\nu = 0$.