Albert-Ludwigs-Universität Freiburg — Winter 2014/15

Particle Physics II

Markus Schumacher, Anna Kopp, Stan Lai

Problem Set X

20 January 2015

In-class exercises

Exercise 50 *Discussion of Concepts for the Weak Interaction*

(a) Helicity, chirality: How are they defined, what are their properties, why are they important?

(b) V-A theory: What does it mean; and how can it be used to describe the weak interaction?

Exercise 51 *Discussion of the Goldhaber Experiment*

The following are only suggestions; we can discuss selected topics from below and/or any other topics. An introduction to the Goldhaber experiment and its role in measuring neutrino helicity is discussed. There is also a lot of information about this experiment on the internet if you want to read about it in more details, including the original paper: Phys. Rev. **109**, 1015 (1958).

Homework

Exercise 52 *Muon decay* **12 Points**

Consider the decay of a muon,

$$\mu^-(p_1) \to e^-(p_4) + \bar{\nu}_e^-(p_2) + \nu_\mu^-(p_3).$$

(a) Draw the Feynman diagram for the leading-order contribution to this process.

(b) Using the Feynman rules for weak interactions, show that the decay amplitude for muon decay can be written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left[\bar{u}_3 \gamma^\mu (1-\gamma^5) u_1\right] \left[\bar{u}_4 \gamma_\mu (1-\gamma^5) v_2\right]$$

with $G_F = \frac{\sqrt{2}}{8}\left(\frac{g_W}{M_W}\right)^2$.

(c) Using the relation

$$\mathrm{Tr}\left[\gamma^\mu(1-\gamma^5)\not{p}_1\gamma^\nu(1-\gamma^5)\not{p}_2\right] \times \mathrm{Tr}\left[\gamma_\mu(1-\gamma^5)\not{p}_3\gamma_\nu(1-\gamma^5)\not{p}_4\right] = 256(p_1\cdot p_2)(p_3\cdot p_4),$$

show that the spin-averaged matrix element is given by:

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 = 64 G_F^2 (p_1\cdot p_2)(p_3\cdot p_4).$$

(d) Show that in the rest frame of the muon,

$$\frac{1}{2}\sum_{\text{spins}} |\mathcal{M}|^2 = 32 G_F^2 m_\mu^2 |\vec{p}_2|(m_\mu - 2|\vec{p}_2|).$$

Here, you are free to make the approximation that $m_e \simeq 0$.

(e) For the muon decay, it can be shown that Fermi's Golden rule leads to the expression

$$d\Gamma = \frac{\sum_{\text{spins}} |\mathcal{M}|^2}{16(2\pi)^4 m_\mu} d|\vec{p}_2| \frac{d^3\vec{p}_4}{|\vec{p}_4|^2}.$$

where the integral over $|\vec{p}_2|$ runs from $\frac{1}{2}m_\mu - |\vec{p}_4|$ up to $\frac{1}{2}m_\mu$, and over $|\vec{p}_4|$ from 0 to $\frac{1}{2}m_\mu$.

Using this information, and integrating out first $|\vec{p}_2|$, and then $d^3\vec{p}_4 = 4\pi|\vec{p}_4|^2 d|\vec{p}_4|$, show that the muon decay rate is given by

$$\Gamma = \frac{G_F^2 m_\mu^5}{24(2\pi)^3}.$$

(f) Use $G_F = 1.166 \times 10^{-5}\ \text{GeV}^{-2}$ to calculate the muon lifetime.

Exercise 53 *Tau lepton lifetime* **2 Points**

The muon lifetime was calculated in exercise 52. Adjust the equation for the width given in exercise 51 (e) to estimate the tau lepton lifetime, assuming it only decays leptonically. Compare the calculated value to the literature value ($\tau_\tau = 2.91 \times 10^{-13} s$).

Hints:
You can neglect the muon mass in comparison to the tau lepton mass.
The calculated value disagrees with the literature value by a factor of about 2.5 which can be explained by the fact that the branching ratio of the tau lepton decays to electron and muon final states is actually only about 2/5.

Exercise 54 *Kinematics of the Goldhaber experiment* **6 Points**

The Goldhaber-Grodznis-Sunyar experiment was discussed during the lecture and consists of the following steps:

- ^{152}Eu decays by electron capture to ^{152}Sm*: $^{152}\text{Eu} + e^- \rightarrow {}^{152}\text{Sm}^* + \nu_e$.
- ^{152}Sm* deexcites by emitting a photon: $^{152}\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma_1$.
- The emitted photon is absorbed by another Sm nucleus: $\text{Sm} + \gamma_1 \rightarrow \text{Sm}^*$.
- This Sm* nucleus immediately deexcites to another photon γ_2 which is captured in the NaI detector.

The key point of the experiment is that γ_1 can only be re-absorbed by forward γ_1 emission of by Sm* in flight due to the recoil energy of the nuclei both in the emission and absorption processes.

(a) Show that γ_1 cannot be reabsorbed by Sm if produced in an Sm* decay at rest.

(b) Show that γ_1 can be absorbed by Sm if emitted in forward direction by Sm* in flight.

You can use the following:

- The energy released in the transition $^{152}\text{Eu} \rightarrow^{152} \text{Sm}^*$ is 911 keV.
- The reconance energy in question (energy difference between the two Sm levels) is 963 keV.
- The resonance width is about 1 eV (note that the natural width is only about 20 meV - the value given is after Doppler broadening due to thermal motion).
- You can approximate the neutrino and photon momentum by the energies released in the transitions in which they are produced (explain why!).
- For the energy-momentum dependence of the nuclei, you can use non-relativistic expressions (explain why!).
- Remember that you need the γ_1 energy in the rest frame of the Sm nucleus to judge whether absorption will take place or not. You should first show that the relation between the photon energy in the frame of the source, E_0, and its energy in another frame (e.g. the lab frame), E, given by a simple Lorentz transformation, is $\frac{E}{E_0} = \sqrt{\frac{1+\beta}{1-\beta}}$ with $v = \beta c$ being the relative speed of the source with respect to the other frame.