Particle Physics II

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Problem Set XII

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In-class exercises

Exercise 60 *Production of Hadrons from Electron-Positron Annihilation at the Z Pole*

Consider electron-positron annihilation ($e^+e^- \to f\bar{f}$) at the Z pole ($E_{\rm CM} \simeq m_Z$).
Note that quark and lepton masses are then negligible, and the photon propagator can be ignored.
In this case, the differential scattering cross-section (try it at home!) can be shown to be:

$$\frac{d\sigma}{d\Omega} = \left(\frac{g_Z^2 E}{16\pi[4E^2 - m_Z^2]}\right)^2 \left([(c_V^f)^2 + (c_A^f)^2][(c_V^e)^2 + (c_A^e)^2][1 + \cos^2\theta] - 8c_V^f c_A^f c_V^e c_A^e \cos\theta\right)$$

(a) Explain why at the Z pole, we have

$$R = \frac{3\sum_q [(c_V^q)^2 + (c_A^q)^2]}{[(c_V^\mu)^2 + (c_A^\mu)^2]}.$$

Recall that R is defined as:

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

(b) What is the value of R in this case? (substitute for c_V^q and c_A^q for the relevant quarks)

Exercise 61 *Number of neutrino flavors*

(a) The total decay width of the Z boson is given by:

$$\Gamma_Z = \Gamma_e + \Gamma_\mu + \Gamma_\tau + \Gamma_{\rm had} + \Gamma_{\rm inv}.$$

Here, Γ_ℓ with $\ell = e, \mu, \tau$ are the partial widths of decays to lepton pairs $\ell\bar{\ell}$, $\Gamma_{\rm had}$ the partial width to decays to quark pairs, and $\Gamma_{\rm inv}$ to invisible particles (like neutrinos).
Show that the ratio of invisible and leptonic decay width is given by

$$R_{\rm inv} = \frac{\Gamma_{\rm inv}}{\Gamma_\ell} = \left[\frac{12\pi R_\ell}{\sigma_{\rm peak}^{\rm had} M_Z^2}\right]^{\frac{1}{2}} - R_\ell - 3.$$

Here, $R_\ell = \frac{\Gamma_{\rm had}}{\Gamma_\ell}$ and $\sigma_{\rm peak}^{\rm had}$ is the cross section for $Z \to$ hadrons for $\sqrt{s} = m_Z$.
You can assume lepton universality; the peak cross section is given by $\sigma_{\rm peak}^{\rm had} = \frac{12\pi\Gamma_e\Gamma_{\rm had}}{M_Z^2\Gamma_Z^2}$.

(b) The theoretical prediction for the Standard model is

$$R_{\rm inv} = N_\nu \left(\frac{\Gamma_\nu}{\Gamma_\ell}\right)_{\rm SM}.$$

with

$$\left(\frac{\Gamma_\nu}{\Gamma_\ell}\right)_{\rm SM} = 1.99125.$$

Show that this implies that the Standard model contains $N_\nu = 3$ light neutrino flavors and calculate the uncertainty on this number (using the uncertainties on the LEP measurements given below). The equations above imply that for larger $\sigma^{\text{had}}_{\text{peak}}$, the predicted number of light neutrino flavors becomes smaller (see Fig. 1).

$$m_Z = (91.1875 \pm 0.0021)\,\text{GeV}$$
$$R_\ell = 20.767 \pm 0.025$$
$$\sigma^{\text{had}}_{\text{peak}} = (41.540 \pm 0.037)\,\text{nb}$$

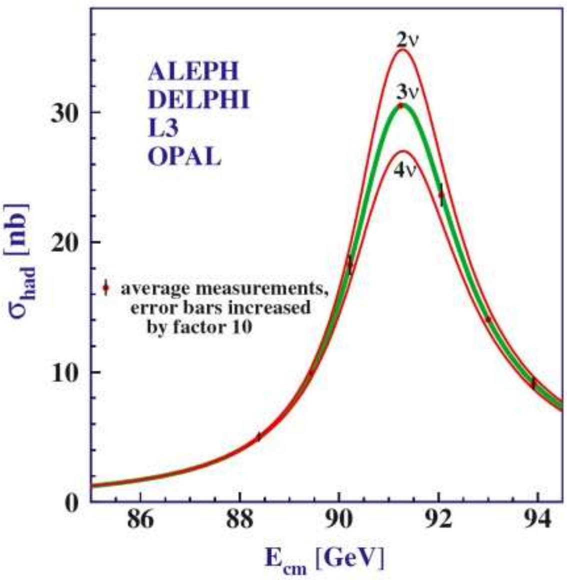

Figure 1: σ(had) as a function of the center-of-mass energy for different numbers of light neutrino flavors.