Albert-Ludwigs-Universität Freiburg Winter 2014/15

Particle Physics II

Markus Schumacher, Anna Kopp, Stan Lai

Problem Set IV

18 November 2014

In-class exercises

Exercise 23 *Completeness of Dirac Equation Solutions*

Given these four solutions to the Dirac Equation:

$$u^{(1)} = N \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \end{pmatrix}, u^{(2)} = N \begin{pmatrix} 0 \\ 1 \\ \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}, v^{(1)} = N \begin{pmatrix} \frac{p_x-ip_y}{E+m} \\ \frac{-p_z}{E+m} \\ 0 \\ 1 \end{pmatrix}, v^{(2)} = -N \begin{pmatrix} \frac{p_z}{E+m} \\ \frac{p_x+ip_y}{E+m} \\ 1 \\ 0 \end{pmatrix},$$

with $N = \sqrt{E+m}$, show that the completeness relation holds for these spinors, namely that:

$$\sum_{s=1,2} u^{(s)}\bar{u}^{(s)} = (\gamma^\mu p_\mu + m), \sum_{s=1,2} v^{(s)}\bar{v}^{(s)} = (\gamma^\mu p_\mu - m).$$

Exercise 24 *Møller Scattering*

Bhabha scattering is name for the QED process $e^+e^- \to e^+e^-$, while Møller scattering is the name for the process $e^-e^- \to e^-e^-$.

(a) Draw the Feynman diagrams for Møller scattering.

(b) What is the relative sign between the two diagrams? (Important for the interference term in the Matrix element squared)

(c) In the lectures, it was shown that the matrix element squared for Bhabha scattering written in terms of Mandelstam variables, can be simplified to:
(after averaging over initial state spins, and summing over the final state spins)

$$|\bar{\mathcal{M}}|^2 = 2e^4 \left[\frac{u^2+t^2}{s^2} + \frac{2u^2}{st} + \frac{u^2+s^2}{t^2} \right].$$

Using crossing symmetry, arrive at the matrix element squared $|\bar{\mathcal{M}}|^2$ for Møller scattering.
(also for averaging over initial state spins and summer over final state spins)

Homework

Exercise 25 *More Møller Scattering* **6 Points**

(a) From the result in Problem 24c, write down the differential cross-section for Møller scattering. You can assume that the electrons are massless (i.e. $\sqrt{s} >> m_e$). Don't forget that you have identical particles in the initial and final states!

(b) Show that if we substitute $\theta \to \pi - \theta$, that the result remains unchanged. Why is that?

Exercise 26 *The Rutherford Limit for Mott Scattering* **7 Points**

Mott scattering entails the process $e^- X \to e^- X$, where X has a mass much larger than the electron energy (could be a muon or a proton). From the lectures, we have calculated the matrix element squared (averaging over inital state spins, and summing over final state spins):

$$|\bar{\mathcal{M}}_{Mott}|^2 = \frac{8e^4}{(p_1 - p_3)^4} \left[(p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) - m_X^2 (p_1 \cdot p_3) - m_e^2 (p_2 \cdot p_4) + 2m_e^2 m_X^2\right].$$

(a) In the rest-frame of the heavy particle X, the heavy particle is stationary before and after the collision ($m_X >> |\vec{p}_e|$). Show the the Mott scattering scross-section in this frame-of-reference is

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \frac{\alpha^2}{4|\vec{p}_e|^4 \sin^4(\theta/2)} \left[m_e^2 + \vec{p}^2 \cos^2(\theta/2)\right].$$

(b) Consider the case where the incident electron is non-relativistic $|\vec{p}_e|^2 << m_e^2$. Show that the differential cross section reduces to the Rutherford scattering formula:

$$\left(\frac{d\sigma}{d\Omega}\right)_{Rutherford} = \frac{\alpha^2}{4m_e^2 v^4 \sin^4(\theta/2)}.$$

Exercise 27 *Threshold Behaviour for $e^+e^- \to \mu^+\mu^-$* **7 Points**

The unpolarized matrix element for $e^+e^- \to \mu^+\mu^-$ or for any fermion-antifermion final state with a mass much larger than the electron mass can be expressed as:

$$|\bar{\mathcal{M}}|^2 = \frac{2e^4}{(p_1 \cdot p_2)^2} \left[(p_1 \cdot p_3)(p_2 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) + m_\mu^2 (p_1 \cdot p_2)\right].$$

Now we want to find the behaviour of the cross-sections for $e^+e^- \to \mu^+\mu^-$ at threshold, so we should not neglect the muon mass in the following:

(a) Express the differential cross-section in terms of the Mandelstam variable $s = E_{CM}^2$ and the relativistic velocity $\beta = \frac{|\vec{p}|}{E}$.

(b) Integrate this to obtain an expression of the total cross-section.