Albert-Ludwigs-Universität Freiburg

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Particle Physics II

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Problem Set V

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In-class exercises

Exercise 28 *Bhabha Scattering*

Bhabha scattering entails the process $e^+e^- \to e^+e^-$.

(a) Draw the Feynman diagrams for this process. How many diagrams are there?

(b) What is the relative sign between the diagrams that you have drawn?

(c) Using Feynman rules for QED, write down the matrix element for this process in terms of spinors and γ-Matrices.

(d) Let's take a crack at looking at the interference term in more detail. In squaring the matrix element let's calculate only the cross-term. Average and sum over initial and final state spins, so you can use the trace theorems to express the cross-term in terms of 4-vectors without any spinors. In evaluating the trace, you can neglect the mass of the electron (relativistic limit).

(e) If you have time, you can multiply out the 4-vectors to get the angular dependence of this cross-term.

Note: of course, if you want to get the cross-section, you cannot ignore the non-cross-terms! We have ignored them here due to lack of time.

Homework

Exercise 29 *Luminosity measurement at LEP* **10 Points**

The measurement of a cross section usually requires the knowledge of the luminosity $\mathcal{L}(t)$. The luminosity relates the recorded event rate and the cross section via

$$\frac{\mathrm{d}\dot{N}}{\mathrm{d}\Omega} = \mathcal{L}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}.$$

For a collider experiment, the luminosity is given as

$$\mathcal{L} = f N_b \frac{N_1 N_2}{4\pi\sigma_x\sigma_y},$$

where f is the orbital frequency of the particle beams, N_b the number of particle bunches, N_1 and N_2 the number of particles in one bunch, and σ_x and σ_y the size of the transverse dimension of the beams at the interaction point.

LEP was an e^+e^- collider with a circumsphere of about 27 km, running at a centre-of-mass energy of $\sqrt{s} = 91.2$ GeV from 1989 to 1995 (in its final year 2000, LEP ran at up to 209 GeV). Both beams contained four bunches with 3×10^{11} particles each. The beams were focused to $\sigma_x = 100\,\mu$m und $\sigma_y = 20\,\mu$m at the interaction points.

(a) What is the unit of luminosity?

(b) Calculate the luminosity at LEP.

(c) The cross section for Z^0 bosons at this energy is about 60 nb. How many such bosons have been produced in a typical year (data-taking for about 10^7 s)?

(d) Measuring the quantities entering the luminosity definition is associated with large uncertainties. For this reason, the LEP experiments measured the luminosity via a reference process, Bhabha scattering at small angles. This cross section is known to very good precision, and at small angles given by

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{\alpha^2}{s}\frac{1}{\sin^4(\theta/2)}.$$

One of the four LEP experiments, OPAL, used special calorimeters (called luminosity monitors) to detect Bhabha scattering events. They were located along the beam pipe in a distance of 2.5 m from the interaction point in both directions and were sensitive in a radial distance to the beam axis of $77 - 127$ mm. How large is the cross section for Bhabha scattering in this sensitive region (the so-called detector acceptance) in units of nb?
Hint: At small angles you can use $\sin\theta \simeq \theta$.

(e) In one year of data-taking, about 800 000 Bhabha events were recorded. At the same time, 17 128 $e^+e^- \to \mu^+\mu^-$ events were recorded. Assuming that 100% of all Bhabha events, and 90% of all muon pair production events were recorded — what is the LEP cross section for muon pair production at $\sqrt{s} = 91.2$ GeV?

(f) Assume the inner distance of the luminosity monitors to the beam pipe is known to an accuracy of 1 mm. How large is the relative uncertainty induced in the luminosity measurement?

Exercise 30 *Compton Scattering* **10 Points**

Compton Scattering describes the process $e^-\gamma \to e^-\gamma$.

(a) From the Feynman rules of QED, obtain the amplitudes for the two leading order diagrams.

(b) Using crossing symmetry and Mandelstam variables, relate the diagrams in the two Compton Scattering scattering amplitudes to those from Electron-Positron Annihilation.

(c) Given that $|\mathcal{M}|^2 = 2e^4(\frac{u}{t} + \frac{t}{u})$ for Electron-Positron Annihilation into a pair of photons, what is the corresponding $|\mathcal{M}|^2$ for Compton Scattering?

(d) Show that the differential cross-section for Compton Scattering in the rest frame of the target electron is given by the Klein-Nishina formula:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{\pi\alpha^2}{m^2}\left(\frac{\omega'}{\omega}\right)^2\left[\frac{\omega'}{\omega} + \frac{\omega}{\omega'} - \sin^2\theta\right],$$

where ω and ω' are the frequencies of the incident and scattered photons, and θ is the angle between the outgoing and incoming photon momenta.
Use the fact that the unpolarized scattering amplitude squared can be written as:

$$|\mathcal{M}|^2 = 2e^4\left[\frac{p\cdot k'}{p\cdot k} + \frac{p\cdot k}{p\cdot k'} + 2m^2\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right) + m^4\left(\frac{1}{p\cdot k} - \frac{1}{p\cdot k'}\right)^2\right],$$

where p, k, k' are the four-momenta of the initial electron, initial photon, and scattered photon, respectively. This can be related to the differential cross-section using:

$$\frac{d\sigma}{d(\cos\theta)} = \frac{(\omega')^2}{32\pi\omega^2 m^2}(|\mathcal{M}|^2).$$

Hint:
Show that

$$\frac{1}{\omega'} - \frac{1}{\omega} = \frac{1}{m}(1 - \cos\theta)$$

is true in the rest frame of the electron.