# Particle Physics II

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# Problem Set VI

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## In-class exercises

**Exercise 31** *Vertex Corrections and the Magnetic Moment*

The interaction of an electron with an electromagnetic field $A_\mu$ has a vertex correction due to next-to-leading order Feynman diagrams. In particular, for small momentum transfer $q^2$, this is given by:

$$e\bar{u}_f\left\{\gamma_\mu\left[1+\frac{\alpha}{3\pi}\frac{q^2}{m_e^2}\left(\ln\frac{m_e}{m_\gamma}-\frac{3}{8}\right)\right]-\frac{\alpha}{2\pi}\frac{1}{2m_e}i\sigma_{\mu\nu}q^\nu\right\}u_i,$$

where $m_e$ is the mass of the electron and $m_\gamma$ is the mass of the virtual photon. The object $\sigma_{\mu\nu}$ is just a compact way to write $\frac{i}{2}[\gamma^\mu,\gamma^\nu]$.

Using the Gordon identity

$$e\bar{u}_f\gamma^\mu u_i=\frac{e}{2m_e}\bar{u}_f\left[(p_f^\mu+p_i^\mu)+i\sigma^{\mu\nu}(p_{\nu,f}-p_{\nu,i})\right]u_i,$$

and equate the term proportional to $i\sigma_{\mu\nu}q^\nu$ to the magnetic moment $\vec{\mu}$ of the electron. Show then that the vertex correction yields the following relation for the gyromagnetic ratio of the electron:

$$\frac{g-2}{2}=\frac{\alpha}{2\pi}.$$

## Homework

**Exercise 32** *Running Coupling Constants in QED* **7 Points**

The dependency on the electromagnetic coupling constant $\alpha_{\text{EM}}$ on the squared momentum transfer $q^2$ is given by:

$$\alpha_{\text{EM}}(q^2) = \frac{\alpha_{\text{EM}}(\mu^2)}{1 - \Pi_{\text{EM}}(q^2, \mu^2)},$$

where $\mu^2$ is a scalar parameter, and $\Pi_{EM}(q^2, \mu^2 = 0)$ is given by:

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \sum_{2m_f < |q|} N_c Q_f^2 \frac{\alpha}{3\pi} (\ln \frac{q^2}{m_f^2} - \frac{5}{3}).$$

Here, $N_c$ stands for the number of different colours for the different fermion species ($N_c = 1$ for fermions while $N_c = 3$ for quarks). The index $f$ runs over the different fermions with charge $Q_f$, and the summation occus for those fermions species with masses that satisfy the condition $2m_f < |q|$ (for which pair creation is possible at the given momentum transfer).

In addition, $\alpha$ is the electromagnetic coupling constant in the low energy limit:

$$\alpha = \alpha_{\text{EM}}(q^2 = 0, \mu) \simeq \frac{1}{137}.$$

(a) For momentum transfers that satisfy $2m_b < |q| < 2m_t$, show that

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \frac{\alpha}{3\pi}(3 + R) \ln \frac{q^2}{m_0^2},$$

where $R = N_c \sum_{f=1}^{5} Q_f^2$ and $m_0 = 0.30$ GeV is the effective mean of all fermion masses in question.

(b) Compare the value of $\alpha_{\text{EM}}$ for momentum transfer $q^2 = M_Z^2$ to the low energy limit $\alpha$.

(c) What value does R take for momentum transfers that allow top quark pair production?

(d) At what momentum transfer does $\alpha_{\text{EM}}$ diverge?
(Note for higher momentum transfers, $m_0 = 0.94$ GeV.)

**Exercise 33** *Running Coupling Constants in QED II* **6 Points**

The OPAL Experiment at the Large Electron-Positron Collider measured the dependency of the coupling constant $\alpha_{EM}$ on the momentum transfer in Bhabha Scattering ($e^+e^- \to e^+e^-$) for very small scattering angles (scattering in the forward direction). The following momentum transfer ranges were investigated:

$$1.81\text{GeV}^2 \leq -t \leq 6.07\text{GeV}^2.$$

Recall that the unpolarized squared scattering amplitude for this process is given by:

$$|\mathcal{M}|^2 = 2e^4 \left( \frac{u^2 + t^2}{s^2} + \frac{2u^2}{st} + \frac{u^2 + s^2}{t^2} \right).$$

(a) Show that the t-channel contribution dominates the scattering process for small angle scattering.

(b) Calculate the effective change of the electromagnetic coupling constant that was measured in the momentum transfer ranges observed:

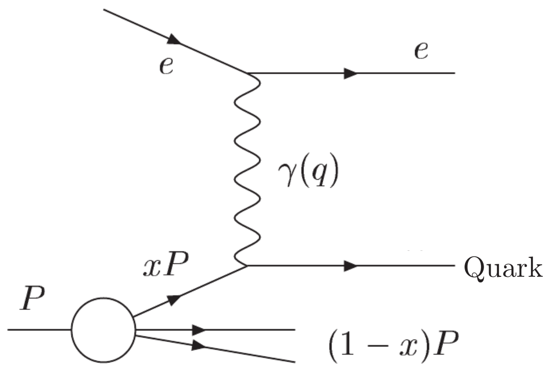
$$\Delta\Pi_{\text{EM}} = \Pi_{\text{EM}}(t_{max}) - \Pi_{\text{EM}}(t_{min}).$$

Recall the relation from the previous problem:

$$\Pi_{\text{EM}}(q^2, \mu^2 = 0) = \sum_{2m_f < |q|} N_c Q_f^2 \frac{\alpha}{3\pi} (\ln \frac{q^2}{m_f^2} - \frac{5}{3}).$$

**Exercise 34** *Kinematics of Deep Inelastic Electron-Proton Scattering* **7 Points**

Deep inelastic electron-proton-scattering can be considered as elastic scattering of an electron with a parton (in the proton).




In the above diagram, the parton carries a fraction $x$ of the proton momentum $P$. Let the momenta of the incoming and outgoing electron be denoted by $k$ and $k'$, respectively, so that the momentum transfer can be written $q = k - k'$.

(a) Show that $x$ satisfies the relation

$$x_{\text{BJORKEN}} = \frac{-q^2}{2P \cdot q}$$

when the transverse momentum of the parton, along with the masses of the partons, electron, and proton in this event can be neglected.

(b) Show that the Lorentz invariant quantity $\nu = \frac{P \cdot q}{M_{proton}}$ is equal to the energy transfer $\tilde{\nu} = E - E'$ of the electron in the rest-system of the proton.

(c) Figure 1 depicts a typical deep inelastic scattering event of an electron with a proton, recorded with the ZEUS Detector at DESY. Here you see the positron coming in from the left with an energy fo 27.5 GeV, and the proton from the right with an energy of 820 GeV. The polar angle is measured with respect to the direction of the proton beam at ZEUS. In this particular event, the electron is scattered at an angle of $\theta_e = 39.3\,^\circ$, and deposits $E'_e = 166\,\text{GeV}$ of energy in the electromagnetic calorimeter.

The following Lorentz-invariant quantities are used to describe the kinematics of deep elastic scattering events.

$$x = \frac{-q^2}{2P \cdot q} \qquad y = \frac{P \cdot q}{P \cdot k} \qquad s = (k+P)^2 \qquad Q^2 = -q^2$$

Derive the relationship between the quantities $Q^2$, $x$, $y$ and $s$. Note that $s$ should be fixed for the collision, so that only two degrees of freedom remain. Neglect also all particle masses.

Calculate the values of $x$ and $Q^2$ for this particular event at ZEUS.

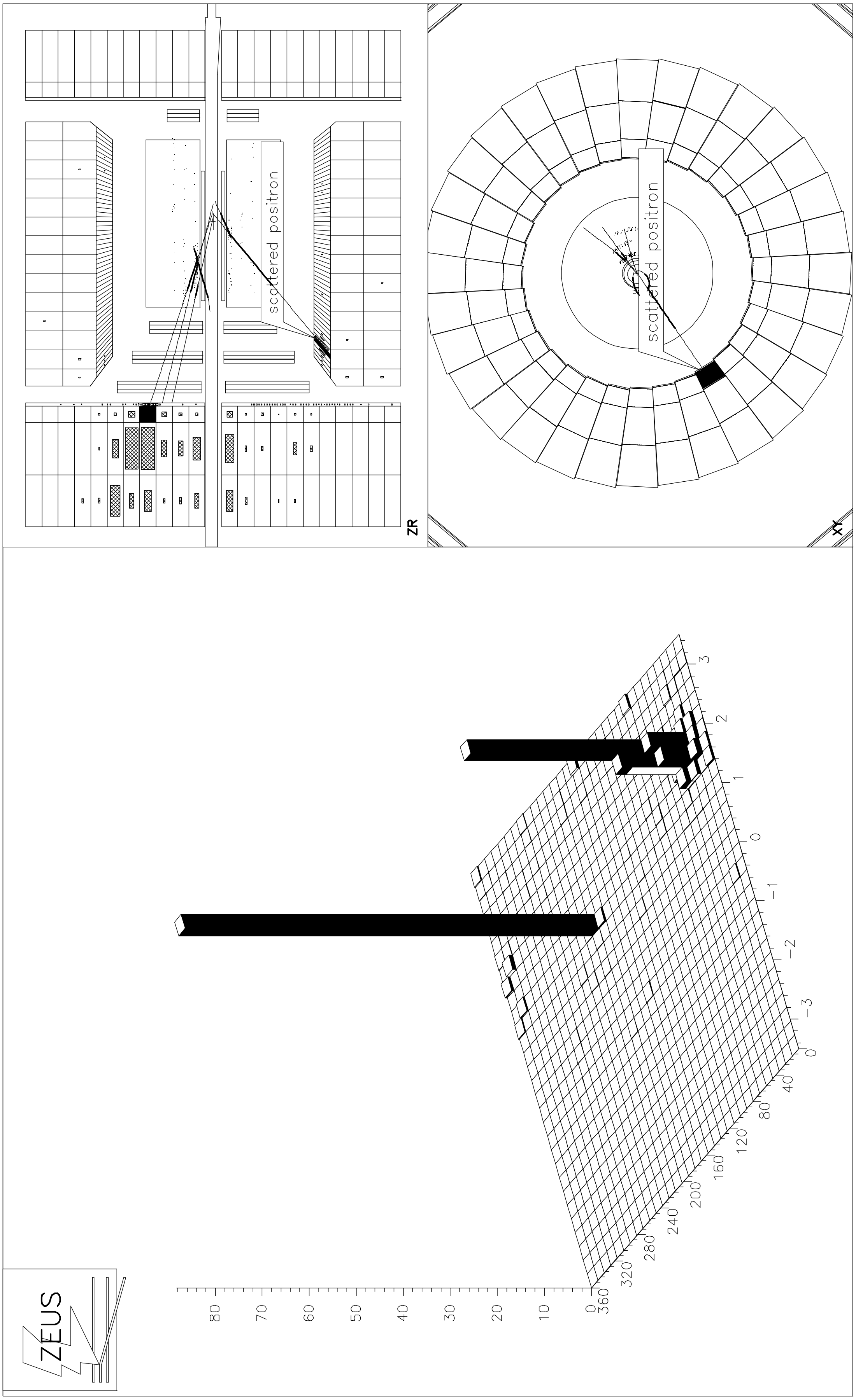



Figure 1: A ZEUS event display of a deep inelastic scattering event shown in an $r$-$z$-projection, an $r$-$\phi$-projection, as well as in the $\eta$-$\phi$-plane. The polar angle is measured with respect to the direction of the proton beam.