Particle Physics II

Markus Schumacher, Anna Kopp, Stan Lai

Problem Set VII

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In-class exercises

Exercise 35 *Conceptual Discussion of Deep Inelastic Scattering*

(a) What's the difference between elastic and inelastic nucleon scattering? How many kinematic variables are necessary to describe the entire process in these two cases?

(b) What does Bjorken-scaling refer to? How can one interpret the results of Bjorken-scaling?

Exercise 36 *Conceptual Discussion of Structure Functions*

From the lectures, one has the following relation for the structure functions of a nucleon:

$$F_2 = x \sum Q_j^2 f_j(x) \qquad \text{and} \qquad F_2 = 2xF_1.$$

Sketch $F_2(x)$ for the following proton "models" as a function of x, without use of mathematical parametrization of the $f_j(x)$ terms.

- The proton consists of a single quark.
- The proton consists of three valence quarks.
- The proton consists of three quarks together in a bound state.
- The proton conists of three bound valence quarks as well as additional sea quarks.

Exercise 37 *Production of Hadrons from Electron-Positron Annihilation at Low Energy*

A good experimental indication that quarks possess a colour charge comes from the measurement of the quantity

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}.$$

For $e^+e^- \to f\bar{f}$ (for any fermion f) scattering at a CM energy E_{CM} which satisfies $m_f \ll E_{\text{CM}} \ll m_Z$, one obtains the following formula for the total cross-section:

$$\sigma = \frac{\pi}{3}\left(\frac{Q_f \alpha}{E}\right)^2,$$

where Q_f is the electric charge of the fermion.

(a) Explain why $R = 3\sum_i Q_i^2$, where i sums over all quark flavours below the E_{CM} threshold.

(b) What is R for low energies (when only the three lightest quarks contribute)? What is R above the c-quark pair threshold and above the b-quark pair threshold?

Compare these results to the sheet attached. (Note the J/ψ is the lightest $c\bar{c}$ resonance, while the Υ is the lightest $b\bar{b}$ resonance.)

Homework

Exercise 38 *Deep inelastic Scatering and the Quark-Parton Model* **8 Points**

The results of deep inelastic ep scattering can be interpreted as elastic scatter processes of the electron with the partons contained in the proton.

(a) The cross section of elastic scattering of an electron off a point-like particle at rest (charge q, spin $\frac{1}{2}$) is given by:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2 q^2}{4E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2} \sin^2\frac{\theta}{2} \right). \tag{1}$$

Here, E and E' are the energies of the incoming and outgoing electron, respectively; θ the scattering angle and $Q^2 = -t$ the virtuality (momentum transfer to the proton).

Explain the meaning of the different terms in equation (1). How does the cross section change for the case of spin-less target particles? How for very heavy target particles?

(b) The elastic scattering of electrons off protons is described by the Rosenbluth equation:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16M^2E^2 \sin^4\frac{\theta}{2}} \frac{E'}{E} \left(K_2(Q^2) \cos^2\frac{\theta}{2} + 2K_1(Q^2) \sin^2\frac{\theta}{2} \right). \tag{2}$$

The quantities $K_1(Q^2)$ and $K_2(Q^2)$ are the form factors known from the lectures. They depend on the charge distribution and the magnetic dipole moment via Fourier transformations. Compare equations (1) and (2) to get the form factors of point-like particles.

(c) Inelastic scatter processes of electrons off protons are described by the differential cross section

$$\frac{d\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{4E^2 \sin^4\frac{\theta}{2}} \left(W_2(Q^2, x) \cos^2\frac{\theta}{2} + 2W_1(Q^2, x) \sin^2\frac{\theta}{2} \right). \tag{3}$$

The structure functions $W_1(Q^2, x)$ and $W_2(Q^2, x)$ depend on Q^2 and the Bjorken variable x. Typically, experiments measure E' and θ. For the theoretical descriptions, the Lorentz invariants Q^2 and x (or ν) are preferred. Let's first get the structure functions $W_1^i(Q^2, x)$ and $W_2^i(Q^2, x)$ for a given quark i in the proton. To do this, start with the ansatz

$$W_{1,2}^i(Q^2, x_i) = \frac{Q_i^2 K_{1,2}^i(Q^2, x_i)}{2m_i Q^2} \delta(x_i - 1). \tag{4}$$

Here, Q_i is the electric charge of the quark being considered (not to be confused with Q, denoting the momentum transfer of the scattering), while m_i is the individual quark mass.

(d) Assume the proton consists of quarks labelled by i with masses m_i, where the quarks carry the fraction z_i of the total momentum p of the proton. What relations exist between the m_i and the total mass M of the proton, and between x_i and x?

(e) The probability distributions of the momentum fractions z_i are typically written $f_i(z_i)$. Derive an expression for the proton structure functions W_1 and W_2 by integrating over z_i and summing over all quarks of the proton, namely,

$$W_{1,2} = \sum_i \int_0^1 dz_i W_{1,2}^i f_i(z_i). \tag{5}$$

Note that you can transform the delta-function $\delta(x_i - 1)$ into an expression that includes a factor $\delta(x - z_i)$.

(f) Experiments show that the dimensionless structure functions

$$F_1(x) = MW_1(Q^2, x) \qquad \text{und} \qquad F_2(x) = \nu W_2(Q^2, x) \tag{6}$$

only depend on the Bjorken variable x (at leading order). This phenomenon is known as Bjorken scaling. Find expressions for $F_1(x)$ and $F_2(x)$, and show that the Callan-Gross relation holds.

Exercise 39 *Elastic Scattering as a Limit of Inelastic Scattering* **8 Points**

The cross section for elastic scattering of an electron off of a point target at rest (with charge q and spin-$\frac{1}{2}$ is given by the Rosenbluth equation (Equation 2) in the previous problem. Inelastic scattering of an electron with a proton is described by Equation 3.

Show that using:

$$W_{1,2}(Q^2, x) = \frac{K_{1,2}(Q^2)}{2MQ^2}\delta(x-1) \tag{7}$$

for the structure functions $W_{1,2}$ together with Equation 3 leads to the Rosenbluth equation for $x = 1$ (elastic scattering).

The following hints might be helpful:

- First take equation 3, substituting for $W_{1,2}(Q^2, x)$ using equation 7.
- Also substitute for x using an appropriate expression in terms of Q^2.
- The delta function $\delta(x-1)$ can undergo variable transformation: $\delta(x-1) = \frac{1}{|f'(Q_0^2)|}\delta(Q^2 - Q_0^2)$, where Q_0^2 is the value of Q^2 that sets the Bjorken variable to $x = 1$.
- Here, $f(Q^2) = x - 1$, where x is expressed in terms of Q^2 while $f'(Q^2)$ is the derivative of $f(Q^2)$ with respect to Q^2.
- Express also dQ^2 in terms of dE', so that you can integrate over dQ^2.
- After a bunch of algebra, you can show that one obtains the result, equation 2.

Exercise 40 *W Boson Decay Branching Ratios and Colour Charge* **4 Points**

You can find the W boson decay branching ratios from the particle data group. The most interesting branching ratios are given below:

- $BR(W \to e\nu) = (10.71 \pm 0.16)\%$
- $BR(W \to \mu\nu) = (10.63 \pm 0.15)\%$
- $BR(W \to \tau\nu) = (11.38 \pm 0.21)\%$

For all intents and purposes, these are almost equal, and you can assume that $BR(W \to \ell\nu) \simeq 11\%$, regardless of lepton flavour. (Why is that so?)

(a) Explain how these measured branching ratios provide evidence that there are 3 colours for colour charge in QCD.

(b) What would the branching ratios for $W \to \ell\nu$ look like if there was only 1 colour charge?

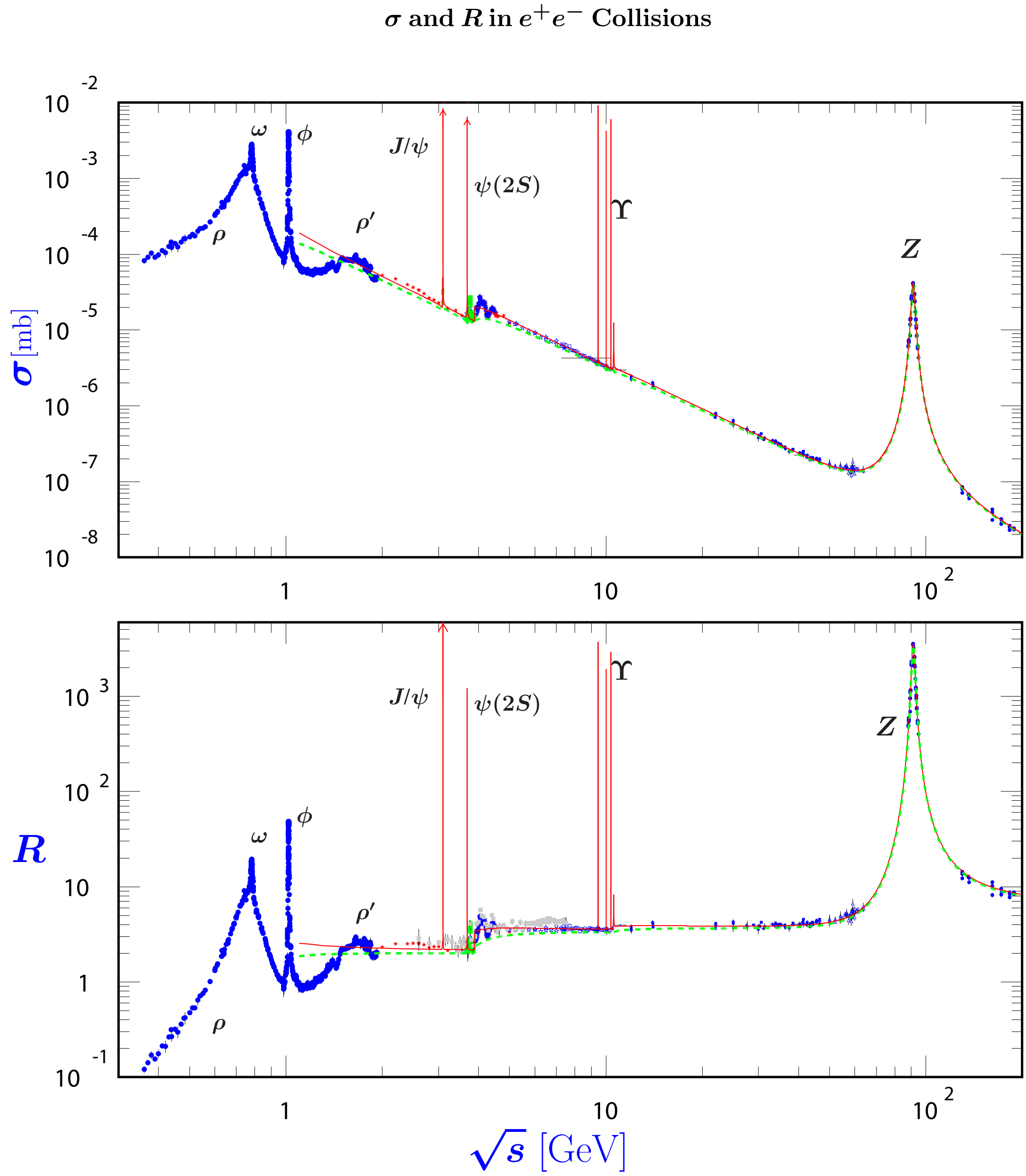

Figure 41.6: World data on the total cross section of $e^+e^- \rightarrow hadrons$ and the ratio $R(s) = \sigma(e^+e^- \rightarrow hadrons, s)/\sigma(e^+e^- \rightarrow \mu^+\mu^-, s)$. $\sigma(e^+e^- \rightarrow hadrons, s)$ is the experimental cross section corrected for initial state radiation and electron-positron vertex loops, $\sigma(e^+e^- \rightarrow \mu^+\mu^-, s) = 4\pi\alpha^2(s)/3s$. Data errors are total below 2 GeV and statistical above 2 GeV. The curves are an educative guide: the broken one (green) is a naive quark-parton model prediction, and the solid one (red) is 3-loop pQCD prediction (see "Quantum Chromodynamics" section of this *Review*, Eq. (9.7) or, for more details, K. G. Chetyrkin *et al.*, Nucl. Phys. **B586**, 56 (2000) (Erratum *ibid.* **B634**, 413 (2002)). Breit-Wigner parameterizations of J/ψ, $\psi(2S)$, and $\Upsilon(nS), n = 1, 2, 3, 4$ are also shown. The full list of references to the original data and the details of the R ratio extraction from them can be found in `[arXiv:hep-ph/0312114]`. Corresponding computer-readable data files are available at `http://pdg.lbl.gov/current/xsect/`. (Courtesy of the COMPAS (Protvino) and HEPDATA (Durham) Groups, May 2010.) See full-color version on color pages at end of book.