Particle Physics II

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Problem Set IX

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In-class exercises

Exercise 46 *Drell-Yan Production in pp Collisions*

Let's have a closer look at Drell-Yan production, which describes the scattering process $pp \to \ell^+\ell^-$, which is mediated by a virtual photon. Here, ℓ refers to a light lepton (e or μ). You know that the parton-level cross-section (where you consider the quarks as initial state particles) can be calculated from QED, and is given by:

$$\sigma_{q\bar{q}\to\ell^+\ell^-} = \frac{1}{N_C} Q_q^2 \frac{4\pi\alpha^2}{3\hat{s}}.$$

Note this expression is shown in terms of $\hat{s} = x_1 x_2 s$, which varies for each pp collision (while s of course stays constant). The factor $N_C = 3$ averages over the 9 different colour configurations $c_i^\dagger c_j$ in the initial state.

(a) Since the variable $\hat{s}$, or alternatively x_1, x_2 vary from collision to collision, find an expression for the differential cross-section

$$\frac{d^2\sigma}{dx_1 dx_2}$$

in terms of the parton distribution functions for the incoming quarks. Don't forget to consider all possible quark flavours.

(b) Relate the variables x_1 and x_2 to the experimental observables $M_{\ell\ell}$ and $y_{\ell\ell}$ (the invariant mass and the rapidity of the combined $\ell^+\ell^-$ system).

(c) Now make a change of variables to give an expression for the differential cross-section in terms of M and y (we omit the obvious subscripts now) instead of x_1 and x_2:

$$\frac{d^2\sigma}{dMdy}.$$

Here you can use the Jacobian transformation:

$$dMdy = Jdx_1dx_2$$

where the Jacobian can be calculated as a 2x2 determinant:

$$J = \begin{vmatrix} \frac{\partial y}{\partial x_1} & \frac{\partial y}{\partial x_2} \\ \frac{\partial M}{\partial x_1} & \frac{\partial M}{\partial x_2} \end{vmatrix}$$

(d) Remove any remaining dependence on the variable s in the expression and show that the differential cross-section $\frac{d\sigma}{dM}$ has a $\frac{1}{M^3}$ dependence.

Homework

Exercise 47 *Hadron Collisions and Rapidity* **12 Points**

The rapidity y is a useful kinematic variable for particles produced in inelastic hadron-hadron collisions (such as $pp \to X$ at the LHC). It is usually expressed as:

$$y = \frac{1}{2} \ln \left(\frac{E + p_L}{E - p_L} \right) \tag{1}$$

where $p_L = p_z$ is the longitudinal momentum along the z-axis (or scattering axis, parallel to the initial particle momentum axis) and E is the energy of the particle in the final state. Other kinematic variables for final state particles include the transverse momentum $p_T = \sqrt{p_x^2 + p_y^2}$ and the mass of the particle m.

Mathematically, the rapidity can also describe Lorentz-transformations along the scattering z-axis with $\beta = p_L/E$:

$$E' = E \cosh y - p_L \sinh y$$

$$p'_L = p_L \cosh y - E \sinh y$$

(a) Show that the relationship $\tanh y = \beta$ holds, by comparing with the standard Lorentz-transformation rules for boosts along the longitudinal z-axis.

(b) Using the results from part (a), confirm that equation (1) holds.

(c) How does the rapidity of a particle change under a Lorentz-boost in the z-direction (as a function of β)? Express your answer in an equation relating y' and y. How does the rapidity difference change between two particles under such a Lorentz-boost?

(d) Which scattering angle and which rapidity correspond to a longitudinal momentum $p_L = 0$? Show also that $y(-p_L) = -y(p_L)$.

(e) Show that the rapidity can also be written as

$$y = \ln \left(\frac{E + p_L}{\sqrt{p_T^2 + m^2}} \right).$$

(f) To determine the maximum and minimum values for the rapidity y in a collision of two particles at energy $E_{CM} = \sqrt{s}$ in the centre-of-mass system, each with mass m. What is the maximum longitudinal momentum that the final state particles can have (in the limit $m \ll \sqrt{s}$)? Show that the maximum and minimum values of the rapidity satisfy:

$$y_{\max} = -y_{\min} = \frac{1}{2} \ln \frac{s}{m^2}.$$

(g) The differential cross section for the production of hadrons for small values of p_L in the final state can be expressed as:

$$d^2\sigma = \pi F_{p_T} dp_T^2 V (dp_L/E),$$

where V is a constant, F_{p_T} varies slowly with p_T (and can be treated as a constant in integration). Find the expression for $\frac{dp_L}{dy}$. Integrate $d^2\sigma$ over p_T^2 and show that the quantity $\frac{d\sigma}{dy}$ is a constant. Draw the distribution of $\frac{d\sigma}{dy}$ as a function of y between $y_{\min}$ and $y_{\max}$.

(h) The average particle multiplicity in the production of hadrons $< n >$ can be obtained by integrating $\frac{d\sigma}{dy}$ over y. How does the average particle multiplicity $< n >$ depend on the centre-of-mass energy $\sqrt{s}$?

(i) Another used quantity in hadron-collider is the pseudorapidity defined as:

$$\eta = -\ln \tan\left(\frac{\theta}{2}\right),$$

where θ is the scattering angle from the longitudinal z-axis. Show that the pseudorapidity and the rapidity are identical for massless particles.

Exercise 48 *Scaling of* α_s **4 Points**

(a) What is the value of α_s at momentum transfers of 10 GeV and 100 GeV, assuming $\Lambda_{QCD} = 300$ MeV. What happens when we consider $\Lambda_{QCD} = 100$ MeV and $\Lambda_{QCD} = 1$ GeV?

(b) What terms describe the regimes $Q^2 \to \infty$ and $Q^2 \to \Lambda^2_{QCD}$? What consequences do these regimes have for the calculation of cross-sections for QCD processes?

Exercise 49 $t\bar{t}$ *Production at the Tevatron and at the LHC* **4 Points**

The dominant production mechanism for top quark production at hadron colliders is through the production of top quark pairs ($t\bar{t}$). This occurs through quark-antiquark annihilation ($q\bar{q} \to t\bar{t}$) or through gluon fusion ($gg \to t\bar{t}$).

(a) Draw the leading order diagrams for the processes $q\bar{q} \to t\bar{t}$ and $gg \to t\bar{t}$.

(b) Assuming that the $t\bar{t}$ pair is produced at threshold (no kinetic energy for the final state particles in the centre-of-mass frame), how large must $\hat{s}$ (the square of the parton-parton centre-of-mass energy) be?

(c) Compute the momentum fraction x for the partons needed to produce a top quark pair $t\bar{t}$ at threshold at the Tevatron ($p\bar{p} \to t\bar{t}$ at $\sqrt{s} = 1.96$ TeV) and at the LHC ($pp \to t\bar{t}$ at $\sqrt{s} = 8$ TeV). Assume that $x = x_1 = x_2$ for this problem.

(d) Which of the two production channels dominate at the Tevatron? Which one at the LHC?